JUNIOR HIGH SCHOOL MATHEMATICS

VOSBURGH AND GENTLEMAN

SECOND COURSE

JUNIOR HIGH SCHOOL MATHEMATICS

SECOND COURSE

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PREFACE

In planning this Second Course in Junior High School Mathematics, the authors have kept the following points in mind:

1. That instruction in mathematics in the eighth school year should continue along the same general lines as in the preceding year.

2. That, from the study of the First Course, the pupil has acquired the following ideas and habits:

(*a*) By checking, the habit of assuming responsibility for the correctness of his results.

(*b*) By estimating his results in advance of the computation, a rational idea of number values.

(*c*) By using the equation in ratios and formulas, some appreciation of its value as a mathematical tool.

(*d*) By measuring and drawing to scale the common geometric figures, some acquaintance with the properties of these figures.

(*e*) By the graphic interpretation and representation of number data, some knowledge of the importance of the graph in science and industry.

(*f*) By emphasizing the idea that *per cent* is a ratio *per hundred*, a comprehension of the fundamentals of percentage.

3. That the course in mathematics should bring the pupil, who leaves school at the end of his eighth school year, in contact with adult activities that lend themselves to mathematical interpretation; and it should afford him

an opportunity for the exercise of his mathematical powers through the handling of a variety of mathematical tools used in the solution of problems of every-day life.

4. That the course should aid the pupil, who continues in school, in deciding whether or not he is capable of continuing his work in mathematics with profit; and it should aid him in acquiring a keener interest in the further study of mathematics. He should get from the course a clear idea of the meaning of mathematics and a vision of its manifold applications to the world's important work.

The attention of teachers is directed to the following features of this course:

1. The nature of the exercises in addition and subtraction, which are especially designed to further remedy the pupil's weakness in handling the difficult combinations, together with the simple forms of checking his work.

2. The combination of common and decimal fractions in the same exercise.

3. The continued use of the equation as a mathematical tool.

4. The further use of the graph in the interpretation of number data, and in the solution of equations.

5. The adherence to business practices in the applications of percentage, and the elimination of such applications as are not likely to fall within the experience of the average person.

6. In Chapters III and IV, the selection of problem material from situations in direct contact with the pupil's home and civic life.

7. The informal presentation of the facts of geometry through observation, construction, and measurement.

8. The extended use of formulas for the mensuration of plane figures and of solids.

9. The emphasis placed upon rational methods of locating the decimal point in multiplication, division, and square root.

10. The introduction of the idea of the degree of accuracy possible in computations dealing with measured data.

11. The gradual development of the solution of the linear equation, and the concise form of explaining each step.

12. The emphasis on independent reasoning in all the mathematical work.

The authors desire to acknowledge their indebtedness to Mr. Henry M. Wright, Head of the Department of Mathematics, English High School, Boston, who is also Chairman of the Council of Mathematics for Intermediate Schools of the city of Boston, and to Mr. Peter F. Gartland, Head Master of the South Boston High School, for their valuable criticisms and suggestions in the preparation of this course.

William Ledley Vosburgh.
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CHAPTER I

REVIEW OF ARITHMETIC

I. Addition and Subtraction

§ 1. Addition by Columns.

To meet the demands of the business world to-day the pupil must form the habit of checking his work.

To check his work he must do two things:

1. Leave a record of the successive steps of the computation.

2. Always go over the computation a second time in some different manner, or order, if possible.

Example	
879,878	(Record of work)
9,876	5 1 √
457,865	5 2 √
76,989	6 0 √
798,657	6 2 √
569,987	4 9 √
498,769	3 2 √
3,292,021	

EXPLANATION. The sum of the units' column is 51; 5 is carried to the next column and the sum 52 is set down under the 51; 5 is carried to the next column and the sum 60 is set down under the 52; 6 is carried to the next column and the sum 62 is set down under the 60; 6 is carried to the next column and the sum 49 is set down under the 62; 4 is carried to the next column and the sum 32 is set down under the 49. In the "Record of work" the sum of two columns is 521, of three 6021, of four 62,021, of five 492,021, of all 3,292,021.

CHECK. If the columns are added upward, check by adding downward. After each column has been added downward and found to be correct, put a check mark (√) by the sum for that column.

Remember that an example added in but one way is only half done. Always *check* your answer.

EXERCISES

Find the sums of each of the following examples. Leave a record of the column sums as shown in the preceding example. Check each answer by the method shown on page 1.

1.	2.	3.	4.
985,697	897,896	798,567	687,698
559,958	987,887	766,957	879,765
867,769	786,865	987,598	566,879
996,878	775,979	659,978	897,987
678,657	868,686	897,697	768,869
765,756	989,798	678,979	689,687
989,877	767,857	957,869	878,876
696,567	667,885	787,957	657,687

5.	6.	7.	8.
967,867	897,687	$8779.96	$5798.58
598,789	988,677	9868.78	4568.69
687,568	879,689	6697.49	7795.98
598,976	697,687	8979.79	8989.45
759,695	877,989	9486.97	5978.77
968,767	696,786	7968.98	9947.87
876,988	769,968	8897.47	9695.56
659,859	688,788	6694.86	7484.49
987,566	697,686	8789.79	8865.64
798,798	779,876	7867.98	6897.88

9.	10.	11.	12.
$1278.76	$13,769.55	$39,954.78	$17,398.83
2759.85	6,358.76	18,868.58	798.89
1397.77	20,978.57	8,497.66	2,179.79
795.69	976.89	15,678.97	11,878.67
1956.78	30,079.86	877.79	798.78
797.89	1,973.54	16,596.73	20,787.58
93.54	398.89	789.95	976.78
2109.76	19,357.85	2,787.78	1,583.89
875.46	597.79	1,976.45	34,578.79
1278.87	30,976.55	387.79	796.59

§ 2. Horizontal Addition. Add the numbers in each line. Check the work by adding the columns and comparing the final totals.

EXERCISES

1.	2.
27 + 56 + 28 = ?	39 + 27 + 39 = ?
36 + 48 + 29 = ?	28 + 39 + 27 = ?
48 + 37 + 27 = ?	27 + 29 + 37 = ?
? + ? + ? = ?	? + ? + ? = ?

3.

$$\begin{array}{r} 27 + 38 + 29 = \ ? \\ 29 + 47 + 56 = \ ? \\ 37 + 35 + 48 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

4.

$$\begin{array}{r} 57 + 59 + 55 = \ ? \\ 58 + 56 + 47 = \ ? \\ 67 + 76 + 49 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

5.

$$\begin{array}{r} 67 + 58 + 75 = \ ? \\ 78 + 87 + 75 = \ ? \\ 66 + 88 + 77 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

6.

$$\begin{array}{r} 88 + 77 + 99 = \ ? \\ 57 + 76 + 67 = \ ? \\ 99 + 98 + 79 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

7.

$$\begin{array}{r} 123 + 542 + 769 = \ ? \\ 254 + 135 + 124 = \ ? \\ 976 + 768 + 857 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

8.

$$\begin{array}{r} 348 + 976 + 535 = \ ? \\ 879 + 798 + 767 = \ ? \\ 545 + 656 + 768 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

9.

$$\begin{array}{r} 786 + 989 + 878 = \ ? \\ 575 + 686 + 785 = \ ? \\ 958 + 567 + 979 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

10.

$$\begin{array}{r} 853 + 789 + 478 = \ ? \\ 976 + 877 + 599 = \ ? \\ 688 + 595 + 678 = \ ? \\ \hline ? + \ ? + \ ? = \ ? \end{array}$$

11. The table at the top of p. 5 gives the attendance in each room of a certain school during the year 1915–1916.

(*a*) Find the total attendance in each room for the year.

(*b*) Find the total attendance in all of the rooms for each quarter.

(*c*) Check the work by adding and comparing the two sets of totals.

Room	1st Quarter	2d Quarter	3d Quarter	4th Quarter	Total
1	205	198	187	202	?
2	205	187	185	191	?
3	199	183	182	193	?
4	198	179	181	185	?
5	203	154	161	169	?
6	187	165	173	182	?
7	175	179	165	176	?
8	193	187	184	193	?
9	185	176	173	182	?
10	194	187	185	194	?
Totals	?	?	?	?	? (check)

12. In the following table, —

(*a*) Find the total number of students in the public and private high schools in the United States in each division as given.

(*b*) Find the total for each column.

(*c*) Check the work by adding and comparing the two sets of totals.

High Schools in the United States, 1915

	Public High Schools		Private High Schools		Total
	Male	Female	Male	Female	
North Atlantic Division	191,507	219,237	28,673	28,597	?
North Central Division	238,615	292,352	16,908	23,589	?
South Atlantic Division	39,898	52,316	10,546	10,342	?
South Central Division	59,523	78,432	11,170	10,342	?
Western Division .	71,901	85,023	5,911	8,559	?
Totals	?	?	?	?	? (check)

13. The following table gives the official report of the potato crop in certain states.

(*a*) Find the total acreage of potatoes in each state for the three years given.

(*b*) Find the total acreage of potatoes for each year in the ten states.

(*c*) Check the work by adding and comparing the two sets of totals.

CROP STATISTICS FOR POTATOES, 1914–1916

STATE	ACREAGE			TOTALS
	1914	1915	1916	
	acres	acres	acres	acres
Maine	156,000	142,000	125,000	?
New Hampshire	17,000	16,000	15,000	?
Vermont	25,000	24,000	23,000	?
Massachusetts	27,000	26,000	25,000	?
Rhode Island	5,000	5,000	5,000	?
Connecticut	24,000	24,000	22,000	?
New York	367,000	355,000	320,000	?
New Jersey	92,000	93,000	85,000	?
Pennsylvania	278,000	280,000	272,000	?
Delaware	11,000	11,000	10,000	?
Totals	?	?	?	? (check)

14. The table at the top of p. 7 gives the official report of the corn crop in certain states.

(*a*) Find the total number of bushels of corn produced in each state for the three years given.

(*b*) Find the total number of bushels produced in the six states for each year.

(*c*) Check the work by adding and comparing the two sets of totals.

UNITED STATES CROP REPORT, DECEMBER, 1916

STATE	PRODUCTION OF CORN (000 omitted)			TOTALS (For three years)
	1914	1915	1916	
	bushels	bushels	bushels	bushels
Iowa	389,424	298,500	366,825	?
Illinois	300,034	374,400	306,800	?
Nebraska	173,950	213,000	192,400	?
Indiana	163,317	190,950	174,658	?
Missouri	158,400	191,750	132,112	?
Texas	124,800	166,850	131,100	?
Totals	?	?	?	? (check)

Other exercises of this kind may be found in encyclopedias, government reports, engineers' handbooks, and so on. Examples of local interest may be obtained from public service corporations, city reports, etc.

§ 3. Subtraction. What is meant by subtrahend? minuend? remainder? See First Course, § 18, p. 15.

EXAMPLE 1

√√√√√√√
3292021
879878
2412143

CHECK

Add upward the answer to the subtrahend, thus:
3 + 8 = 11, put check (√) over 1; carry 1 to 4
5 + 7 = 12, put check (√) over 2; carry 1 to 1
2 + 8 = 10, put check (√) over 0; etc.

EXERCISES

1–8. From the sum of Ex. 1, page 2, in addition, take away each addend separately. Check each answer as shown above.

9. From 420,720 take 198,254.

10. From 210,031,201 take 59,763,789.

11. Subtract \$87,578,756 from \$201,110,000.

12. Find the difference between \$301,201,000 and \$89,563,974.

13. Take 37,776,677 from 100,030,231.

14. Find the excess of \$110,000,011 over \$89,789,709.

15. From \$201,010,003 take \$97,578,357.

16. By how much does the area of Texas (265,896 sq. mi.) exceed the combined areas of New England (66,424 sq. mi.), New York (49,204 sq. mi.), and Pennsylvania (45,126 sq. mi.)?

17. In the following table, —

(*a*) Find the increase of each item for 1915.

(*b*) Find the totals of the columns.

(*c*) Check by comparing the increase of the totals.

BELL TELEPHONE SYSTEM IN THE UNITED STATES

Combined Balance Sheets, 1914 and 1915

	DEC. 31, 1914	DEC. 31, 1915	INCREASE
Telephone plant .	\$847,204,803	\$880,068,520	?
Supplies, tools, etc.	15,701,601	15,951,582	?
Receivables . . .	37,275,586	43,518,625	?
Stocks and bonds .	69,800,814	72,652,646	?
Totals	?	?	? (check)

II. Fractions

§ 4. Review of Fractions.

EXERCISES

A. In each of the following exercises, find the sum:

1. $\frac{1}{2} + \frac{3}{5} + \frac{2}{3} + \frac{3}{4}$
2. $3\frac{2}{3} + 7\frac{8}{9} + 5\frac{7}{12}$
3. $\frac{2}{5} + .3 + \frac{1}{4} + \frac{1}{2}$
4. $\frac{2}{3} + .25 + .375$
5. $.875 + \frac{3}{4} + \frac{1}{8} + .37$
6. $\frac{7}{8} + .625 + \frac{5}{6} + .25$
7. $\frac{3}{8} + .72 + \frac{4}{5} + .43$
8. $9\frac{1}{4} + 17.375 + .6256$
9. $12.7 + 13.9 + 14\frac{4}{5} + 15\frac{3}{5}$
10. $27\frac{1}{2} + 87.7 + 39\frac{1}{4}$
11. $87.5 + 37\frac{1}{4} + 26\frac{7}{8}$
12. $95\frac{5}{8} + 87.3 + 56\frac{2}{3}$
13. $78\frac{4}{5} + 56\frac{3}{4} + 77.35$
14. $9\frac{2}{3} + 7\frac{1}{2} + 19.25$
15. $8\frac{7}{16} + 9.625 + 7.70$
16. $5.65 + 7\frac{1}{2} + 13\frac{2}{3}$

B. In each of the following exercises, find the difference:

1. $23\frac{1}{4} - 3.73$
2. $16.25 - 7\frac{3}{8}$
3. $30\frac{7}{8} - 19.9$
4. $31.75 - 19\frac{2}{3}$
5. $81\frac{5}{6} - 37.7$
6. $20 - .87\frac{1}{2}$
7. $13.25 - \frac{5}{6}$
8. $91.37 - 19\frac{7}{8}$
9. $71.25 - 29\frac{5}{8}$
10. $201.53 - 78\frac{5}{6}$

C. In each of the following exercises, find the product:

1. $\frac{7}{12} \times \frac{16}{25} \times \frac{3}{7}$
2. $27\frac{1}{4} \times 8\frac{1}{3}$
3. $14\frac{2}{3} \times 13\frac{3}{4}$
4. $15\frac{1}{4} \times .5$
5. $12.25 \times 1\frac{1}{3}$
6. $13\frac{3}{4} \times 16\frac{5}{6}$
7. $18\frac{7}{8} \times 25\frac{5}{6}$
8. $39\frac{5}{8} \times 64\frac{1}{6}$
9. $73.71 \times 8\frac{4}{5}$
10. $29\frac{5}{8} \times 7.7$
11. $85\frac{3}{8} \times 56\frac{4}{5}$
12. $79.31 \times 3\frac{4}{5}$
13. $87.78 \times 9\frac{3}{4}$
14. $93\frac{5}{6} \times 8.5$
15. 96.25×7.75
16. $45\frac{3}{4} \times 16.2$

D. In each of the following exercises, find the quotient:

1. $.75 \div \frac{3}{5}$
2. $19\frac{1}{2} \div 3$
3. $42\frac{3}{5} \div 5$
4. $84.5 \div \frac{1}{2}$
5. $73.25 \div .8$
6. $3.75 \div \frac{3}{4}$
7. $25\frac{5}{8} \div 1\frac{1}{2}$
8. $91.15 \div \frac{3}{8}$
9. $83.75 \div 1\frac{5}{8}$
10. $47.625 \div 3\frac{3}{4}$
11. $79.875 \div 3\frac{1}{2}$
12. $112.05 \div 3\frac{1}{4}$
13. $215.04 \div 3\frac{8}{25}$
14. $373.73 \div 6.9$
15. $1567.87 \div .79$
16. $85.47 \div .23$
17. $76.56 \div .083$
18. $87.27 \div .097$
19. $245.8 \div .92$
20. $516.2 \div .009$

E. Drill Table — Common and Decimal Fractions

1.	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{12}$
2.	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{5}{9}$	$\frac{7}{10}$	$\frac{5}{12}$
3.	.1	.2	.3	.4	.5	.6	.7	.8	.9	.10
4.	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{7}{6}$	$\frac{8}{7}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{11}{10}$	$\frac{13}{12}$
5.	.20	.25	.30	.40	.50	.60	.70	.75	.80	.90
6.	$.12\frac{1}{2}$	$.16\frac{2}{3}$	$.33\frac{1}{3}$	.08	$.66\frac{2}{3}$	$.37\frac{1}{2}$	$.62\frac{1}{2}$	$.87\frac{1}{2}$	$.83\frac{1}{3}$	$.00\frac{1}{2}$

Suggested drills with this table:

(*a*) Read the fractions in each of the lines 1, 2, and 4 as decimals.

(*b*) Read the decimals in each of the lines 3, 5, and 6 as common fractions, reduced to lowest terms.

(*c*) Add, by columns, lines 1 and 2; lines 2 and 4; lines 1 and 4.

(*d*) Add, by columns lines 3 and 5; lines 5 and 6; lines 3 and 6.

(*e*) Add, by columns, lines 1 and 3; lines 2 and 3; lines 2 and 5.

(*f*) See if you can find the sum of all fractions in each column.

§ 5. Economy in Multiplying with the Fractions in the Drill Table.

EXAMPLE 1. Multiply 348 by .75.

$$.75 = \frac{3}{4};\ \text{hence, } \frac{3}{\cancel{4}} \text{ of } \overset{87}{\cancel{348}} = 261.$$

EXAMPLE 2. Multiply 1728 by $.16\frac{2}{3}$.

$$.16\frac{2}{3} = \frac{1}{6};\ \text{hence, } \frac{1}{\cancel{6}} \text{ of } \overset{288}{\cancel{1728}} = 288.$$

EXERCISES

In each of the following exercises, find the product:

1. $144 \times .33\frac{1}{3}$
2. $96 \times .25$
3. $142 \times .50$
4. $220 \times .20$
5. $268 \times .12\frac{1}{2}$
6. $184 \times .25$
7. $760 \times .10$
8. $240 \times .16\frac{2}{3}$
9. $312 \times .33\frac{1}{3}$
10. $560 \times .12\frac{1}{2}$
11. $1000 \times .25$
12. $1200 \times .16\frac{2}{3}$
13. $1500 \times .33\frac{1}{3}$
14. $84 \times .75$
15. $96 \times .66\frac{2}{3}$
16. $48 \times .37\frac{1}{2}$
17. $64 \times .87\frac{1}{2}$
18. $72 \times .66\frac{2}{3}$
19. $120 \times .75$
20. $96 \times .62\frac{1}{2}$
21. $160 \times .37\frac{1}{2}$
22. $240 \times .37\frac{1}{2}$
23. $248 \times .75$
24. $225 \times .60$
25. $126 \times .83\frac{1}{3}$
26. $216 \times .83\frac{1}{3}$
27. 84×1.25
28. 96×1.50
29. $72 \times 1.33\frac{1}{3}$
30. 120×1.75
31. 156×1.75
32. 196×1.25
33. $150 \times 1.66\frac{2}{3}$
34. 315×1.20
35. $440 \times 1.12\frac{1}{2}$
36. 640×1.50
37. $1256 \times 1.37\frac{1}{2}$
38. $1728 \times 1.87\frac{1}{2}$
39. 1760×1.75
40. $3030 \times 1.66\frac{2}{3}$

41. 2416×1.25
42. $5280 \times 1.12\frac{1}{2}$
43. $4221 \times 1.33\frac{1}{3}$
44. 2150×1.50
45. $5040 \times 1.66\frac{2}{3}$
46. $31.23 \times .33\frac{1}{3}$
47. $42.57 \times .66\frac{2}{3}$
48. 61.6×1.25
49. 72.8×1.50
50. 8.28×1.75

§ 6. Aliquot Parts of 10 and 100.

An *aliquot part* of a number is a number that is contained in it an integral number of times.

For example, 2, $2\frac{1}{2}$, and 5 are aliquot parts of 10; 20, 25, and $33\frac{1}{3}$ are aliquot parts of 100.

TABLE OF ALIQUOT PARTS

PART	NUMBER	
	10	100
$\frac{1}{10}$	1	10
$\frac{1}{5}$	2	20
$\frac{1}{2}$	5	50
$\frac{1}{4}$	$2\frac{1}{2}$	25
$\frac{1}{8}$	$1\frac{1}{4}$	$12\frac{1}{2}$
$\frac{1}{3}$	$3\frac{1}{3}$	$33\frac{1}{3}$
$\frac{1}{6}$	$1\frac{2}{3}$	$16\frac{2}{3}$

You should become familiar with this table.

§ 7. Multiplication by Aliquot Parts of 10 and 100.

To multiply a number by 10 *or* 100, *you annex the required number of zeros, if it is a whole number; if it is a decimal, you move the point the required number of places to the right.* For example, $582 \times 10 = 5820$; $4.8 \times 10 = 48$; $87.6 \times 100 = 8760$; etc.

To multiply a number by an aliquot part of 10 *or* 100, *you first multiply by* 10 *or* 100; *then take the fractional part of this product.* For example, $84 \times 16\frac{2}{3} = 84 \times \frac{100}{6} = 1400$; $68 \times 2\frac{1}{2} = \frac{68 \times 10}{4} = 170$.

NOTE. When the division will result in an integer, it is easier to divide first and then multiply. For example, $84 \times 16\frac{2}{3} = \frac{1}{6}$ of $84 \times 100 = 1400$.

EXERCISES

In each of the following exercises, find the product:

1. $72 \times 12\frac{1}{2}$
2. $96 \times 12\frac{1}{2}$
3. $102 \times 33\frac{1}{3}$
4. $156 \times 33\frac{1}{3}$
5. $84 \times 12\frac{1}{2}$
6. $144 \times 16\frac{2}{3}$
7. $156 \times 12\frac{1}{2}$
8. $196 \times 33\frac{1}{3}$
9. $7.2 \times 12\frac{1}{2}$
10. $8.4 \times 16\frac{2}{3}$
11. $12.56 \times 16\frac{2}{3}$
12. $\$24.80 \times 12\frac{1}{2}$
13. $\$12.80 \times 16\frac{2}{3}$
14. $\$16.60 \times 33\frac{1}{3}$
15. $\$36.30 \times 16\frac{2}{3}$
16. $\$56.70 \times 33\frac{1}{3}$
17. $\$535.50 \times 12.5$
18. $\$2028.50 \times 2.5$
19. $\$3060 \times 8\frac{1}{3}$
20. $\$360 \times 1\frac{2}{3}$

§ 8. Multiplication by Aliquot Parts of $1.00.

When the multiplicand and multiplier are both abstract numbers, you may interchange them; when one of them is concrete, you may disregard this fact until you desire to label your answer. By doing this, you may often save labor in multiplying when you are finding the cost of articles purchased at a fractional part of a dollar for each article. For example, $96 \times 33\frac{1}{3}¢ = \frac{1}{3}$ of $96 \times \$1 = \frac{1}{3}$ of $\$96 = \$32$; $144 \times 16\frac{2}{3}¢ = \frac{1}{6} \times \$144 = \$24$.

EXERCISES

In each of the following exercises, find the cost of each article and the total cost:

1. 28 lb. at 25¢
   96 lb. at 25¢
   112 lb. at 25¢
   144 lb. at 25¢
   216 lb. at 25¢
2. 96 lb. at $33\frac{1}{3}¢$
   108 lb. at $33\frac{1}{3}¢$
   156 lb. at $33\frac{1}{3}¢$
   228 lb. at $33\frac{1}{3}¢$
   522 lb. at $33\frac{1}{3}¢$

3. 84 lb. at $16\frac{2}{3}¢$
114 lb. at $16\frac{2}{3}¢$
240 lb. at $16\frac{2}{3}¢$
426 lb. at $16\frac{2}{3}¢$
612 lb. at $16\frac{2}{3}¢$

4. 96 yd. at $12\frac{1}{2}¢$
128 yd. at $12\frac{1}{2}¢$
256 yd. at $12\frac{1}{2}¢$
412 yd. at $12\frac{1}{2}¢$
648 yd. at $12\frac{1}{2}¢$

5. 712 yd. at 50¢
432 yd. at 25¢
804 yd. at $33\frac{1}{3}¢$
512 yd. at $12\frac{1}{2}¢$
336 yd. at $16\frac{2}{3}¢$

6. 2156 yd. at 25¢
1002 yd. at $33\frac{1}{3}¢$
1256 yd. at $12\frac{1}{2}¢$
782 yd. at 10¢
990 yd. at $16\frac{2}{3}¢$

7. 78.5 yd. at 10¢
84.75 yd. at $33\frac{1}{3}¢$
68.25 yd. at 20¢
76 yd. at 25¢
114 yd. at $16\frac{2}{3}¢$

8. 112.2 yd. at $33\frac{1}{3}¢$
287.4 yd. at $16\frac{2}{3}¢$
178.4 yd. at $12\frac{1}{2}¢$
256.0 yd. at $12\frac{1}{2}¢$
310.4 yd. at 10¢

9. 96 lb. at $3\frac{1}{3}¢$
72 lb. at $6\frac{1}{4}¢$
96 lb. at $6\frac{1}{4}¢$
89 lb. at 5¢
76 lb. at $2\frac{1}{2}¢$

10. 312 lb. at $2\frac{1}{2}¢$
256 lb. at 5¢
264 lb. at $3\frac{1}{3}¢$
560 lb. at $6\frac{1}{4}¢$
640 lb. at $1\frac{1}{2}¢$

NOTE. In the following exercises it will be necessary to interchange multiplicand and multiplier. For example, the cost of 50 lb. at 42¢ gives the same amount as the cost of 42 lb. at 50¢.

11. 50 lb. at 42¢
25 lb. at 88¢
$16\frac{2}{3}$ lb. at 72¢
$12\frac{1}{2}$ lb. at 56¢
$33\frac{1}{3}$ lb. at 57¢

12. 25 bu. at 96¢
50 bu. at 105¢
125 bu. at 108¢
250 bu. at 112¢
$12\frac{1}{2}$ bu. at 112¢

13. 250 bu. at 85¢
125 bu. at 91¢
500 bu. at 93¢
25 bu. at 85¢
$12\frac{1}{2}$ bu. at 91¢

14. 84 lb. at $62\frac{1}{2}$¢
75 lb. at 76¢
$37\frac{1}{2}$ lb. at 64¢
$87\frac{1}{2}$ lb. at 56¢
125 lb. at 60¢

§ 9. Division by Aliquot Parts of 10 and 100.

To divide a number by 10 *or* 100, *you move the decimal point the required number of places to the left.* For example, $56 \div 10 = 5.6$; $78.2 \div 10 = 7.82$; $7820 \div 100 = 78.2$.

To divide a number by an aliquot part of 10 *or* 100, *you first divide by* 10 *or* 100; *then multiply this quotient by the equivalent fraction inverted.* For example,

$$96 \div 25 = 96 \div \tfrac{100}{4} = \tfrac{96}{100} \times \tfrac{4}{1} = .96 \times 4 = 3.84.$$

EXERCISES

A. In each of the following exercises, find the quotient:

1. $15 \div 2\frac{1}{2}$
2. $25 \div 2\frac{1}{2}$
3. $250 \div 2\frac{1}{2}$
4. $125 \div 2\frac{1}{2}$
5. $125 \div 12\frac{1}{2}$
6. $85 \div 12\frac{1}{2}$
7. $250 \div 12\frac{1}{2}$
8. $500 \div 12\frac{1}{2}$
9. $1000 \div 12\frac{1}{2}$
10. $100 \div 12\frac{1}{2}$
11. $10 \div 12\frac{1}{2}$
12. $100 \div 33\frac{1}{3}$
13. $240 \div 75$
14. $900 \div 33\frac{1}{3}$
15. $720 \div 16\frac{2}{3}$

B. In each of the following exercises, find the quantity:

	Cost	Price per Article		Cost	Price per Article
1.	\$12	$33\frac{1}{3}$¢	**9.**	\$450	$33\frac{1}{3}$¢
2.	\$56	25¢	**10.**	\$315	$16\frac{2}{3}$¢
3.	\$96	$16\frac{2}{3}$¢	**11.**	\$600	75¢
4.	\$72.50	$12\frac{1}{2}$¢	**12.**	\$700	$87\frac{1}{2}$¢
5.	\$15.25	$2\frac{1}{2}$¢	**13.**	\$3030	$37\frac{1}{2}$¢
6.	\$12.10	$3\frac{1}{3}$¢	**14.**	\$2160	$62\frac{1}{2}$¢
7.	\$4.20	$1\frac{1}{4}$¢	**15.**	\$4500	$66\frac{2}{3}$¢
8.	\$112.25	$12\frac{1}{2}$¢			

MISCELLANEOUS APPLIED PROBLEMS

1. In the state of Washington 6,873,000 pounds of hops were produced in 1916. There was an average yield of 1525 pounds to the acre. How many acres of hops were planted?

2. The apple crop in the U. S. for 1916 amounted to 67,695,000 barrels; of these 45,795,000 barrels were winter varieties. How many barrels were there of other varieties?

3. The sugar crop of Cuba for 1916 was estimated at 3,008,000 tons. If the average load of a vessel was 645 tons, how many loads were there?

4. The horse-radish crop of 1909 covered 1475 acres and was valued at $233,885. What was the average value of the crop on one acre?

5. The receipts of seven markets in the U. S. for September, 1916, were 59,010,779 pounds of butter; in September, 1915, 62,550,550 pounds. What was the decrease in 1916?

6. In these markets 814,533 cases of eggs were received in September, 1916; in September, 1915, 836,778 cases were received. What was the decrease in 1916?

7. In 1915, 59,898,000 acres produced 1,011,505,000 bu. of wheat. Find to the nearest bushel the average production per acre.

8. In the U. S., in 1914, 3,711,000 acres produced 409,821,000 bushels of potatoes. What was the average production in bushels per acre?

9. The estimated population of the 48 states of the United States on January 1, 1917, was 102,459,610. What was the average population?

10. In 1914, 476,000 acres in Canada produced 85,672,000 bu. of potatoes. What was the average production, in bushels per acre?

11. The number of miles covered by an automobile on each day for five days was as follows: 190.4 mi., 162.5 mi., 150.8 mi., 173.4 mi., 182.4 mi. Find the total number of miles covered and the average number traveled a day.

12. The record of 6 loads of coal delivered was as follows: 2800 lb., 2625 lb., 2860 lb., 3035 lb., 2950 lb., 2980 lb. Find in pounds the total amount delivered and the cost at \$9.50 a ton (2000 lb.).

13. A merchant bought 120 baskets of peaches at $62\frac{1}{2}¢$ a basket. He sold $\frac{4}{5}$ of them at \$1.25 a basket, the rest at 90¢ a basket. What was his gain?

14. A owns a farm containing 175 acres; .2 of it is pasture land worth \$28 an acre; the remainder is meadow, worth \$75 an acre. What is the value of the entire farm?

15. The cost of producing an acre of corn is estimated as follows: Seed, 28¢; plowing and harrowing, \$3.25; planting, 45¢; cultivating, \$2.75; harvesting, \$1.65. What is the total cost? At this rate, what would it cost to produce 25 acres of corn?

16. A bushel of wheat occupies about $\frac{5}{4}$ cu. ft. of space. How many bushels can be stored in a bin that contains 246 cu. ft.?

17. A cubic foot of stove coal weighs 54.5 lb. What is the weight of coal in a bin containing 28 cu. ft.? Express your answer in tons.

18. The freight rate between two cities is $37\frac{1}{2}¢$ per 100 lb. What must be paid for 2760 lb. of freight between these two cities?

19. Corn weighs 56 lb. per bushel. At $87\frac{1}{2}$¢ a bushel, find the cost of 1400 lb. of corn.

20. If cream contains .03 protein, .26 fat, .028 carbohydrates, .018 salts, and the rest water, find the amount of water. How much water is there in an 8-quart can of cream?

21. For each of the following states and for the United States, find the average population per square mile (to one decimal place).

	Area (In sq. mi.)	Population (Est. Jan. 1, 1917)
(*a*) California	158,297	2,983,800
(*b*) Connecticut	4,965	1,254,900
(*c*) Illinois	56,665	6,193,600
(*d*) Louisiana	48,506	1,843,000
(*e*) Maine	33,040	774,900
(*f*) Massachusetts	8,266	3,747,600
(*g*) Missouri	69,420	3,420,100
(*h*) Nevada	110,690	108,700
(*i*) New Jersey	8,224	2,981,100
(*j*) New York	49,204	10,366,700
(*k*) Ohio	41,040	5,181,200
(*l*) Pennsylvania	45,126	8,591,000
(*m*) Rhode Island	1,248	620,100
(*n*) Texas	265,896	4,472,500
(*o*) Virginia	42,627	2,202,500
(*p*) Washington	69,127	1,565,800
(*q*) Wisconsin	56,066	2,513,800
(*r*) United States	3,743,306	113,309,300

CHAPTER II

PERCENTAGE

I. Per Cent Relation

§ 10. Review. ***Per cent*** means the number per hundred, or the number of hundredths.

EXERCISES

1. In the following table, find what per cent each number is of every other number in the same row, or column. *Give complete statements.*

8	10	12	16	100
16	20	24	32	200
24	40	48	64	500
32	60	72	80	1000

Thus: 8 is $\frac{1}{2}$, or 50 %, of 16; 8 is $\frac{4}{5}$, or 80 %, of 10; etc.

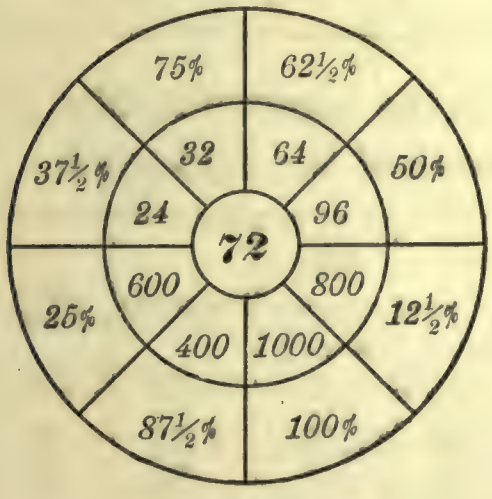


Fig. 1.

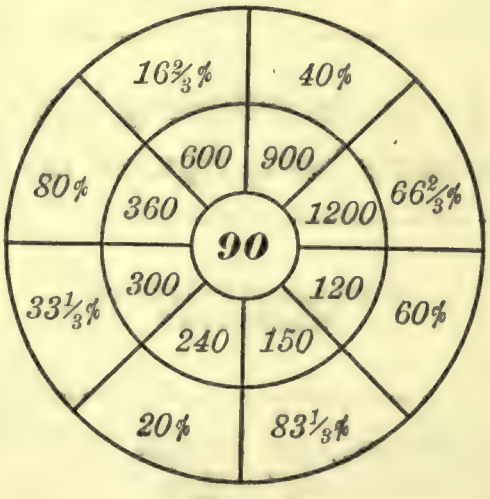


Fig. 2.

2. Find the per cents of the numbers within the circle (Fig. 1). *Give complete statements.*

3. Find the per cents of the numbers within the circle (Fig. 2). *Give complete statements.*

4. In the following table, find the per cents indicated of each number in the middle line. *Give complete statements.*

1%			2 %			3%			5%		
20	60	80	100	120	160	200	300	800	1000	5000	7000
$\frac{1}{2}$%			$\frac{1}{5}$%			$\frac{1}{8}$%			$\frac{1}{10}$%		

5. Memorize the following table.

Fractional parts of the whole expressed as hundredths and as per cents.

$\frac{1}{10} = .10 = 10\%$	$\frac{3}{10} = .30 = 30\%$
$\frac{1}{8} = .12\frac{1}{2} = 12\frac{1}{2}\%$	$\frac{2}{3} = .66\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{6} = .16\frac{2}{3} = 16\frac{2}{3}\%$	$\frac{3}{4} = .75 = 75\%$
$\frac{1}{5} = .20 = 20\%$	$\frac{5}{6} = .83\frac{1}{3} = 83\frac{1}{3}\%$
$\frac{1}{4} = .25 = 25\%$	$\frac{3}{8} = .37\frac{1}{2} = 37\frac{1}{2}\%$
$\frac{1}{3} = .33\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{5}{8} = .62\frac{1}{2} = 62\frac{1}{2}\%$
$\frac{1}{2} = .50 = 50\%$	$\frac{7}{8} = .87\frac{1}{2} = 87\frac{1}{2}\%$

6. In the following table, fill in the required results, the per cents being figured on the basis of the 1915 price.

Wholesale Prices of Commodities	1915 Nov. 1	1916 Nov. 1	Increase	Per Cent
(*a*) Wheat, per bushel . . .	$1.25	$1.85	$ .60	48
(*b*) Corn, per bushel	.76	1.15	.39	51
(*c*) Oats, per bushel	.415	.5825	?	?
(*d*) Barley, per bushel . . .	.64	1.23	?	?
(*e*) Rye, per bushel	1.08	1.51	?	?
(*f*) Flour, per barrel . . .	5.20	8.25	?	?
(*g*) Potatoes, per 100 lb. . .	2.50	5.50	?	?
(*h*) Apples, per barrel . . .	2.50	2.75	?	?

7. In the following table fill in the required results, the per cents being figured on the basis of the populations in 1910.

City	Population (1910)	Population (Est., 1917)	Increase (Per Cent)
(*a*) Boston	671,000	768,000	?
(*b*) Detroit	466,000	825,000	?
(*c*) Los Angeles . .	319,000	600,000	?
(*d*) New York . . .	4,767,000	5,670,000	?

8. In the following table, fill in the required results, where the per cent to be found is the per cent won on the basis of the total games played.

National League Records for 1917

Clubs	Games Won	Games Lost	Per Cent (to nearest tenth)
(*a*) New York	98	56	63.6
(*b*) Philadelphia . . .	87	65	?
(*c*) St. Louis	82	70	?
(*d*) Cincinnati . . .	78	76	?
(*e*) Chicago	74	80	?
(*f*) Boston	72	81	?
(*g*) Brooklyn	70	81	?
(*h*) Pittsburgh . . .	51	103	?

Specimen Solution. (*New York.*)

$98 + 56 = 154$ (games played).

$\frac{98}{154}$ = fractional part won.

$\frac{98}{154}$ = what per cent won?

$\frac{98}{154} = .6363$ or 63.6%. *Ans.* 63.6%.

Note. To get the result correct to the nearest tenth of one per cent, the quotient must be found to four decimal places.

9. In the following table, fill in the required results:

AMERICAN LEAGUE RECORDS FOR 1917

Clubs	Games Won	Games Lost	Per Cent (to nearest tenth)
(*a*) Chicago	100	54	?
(*b*) Boston	90	62	?
(*c*) Cleveland	88	66	?
(*d*) Detroit	78	75	?
(*e*) Washington . . .	74	79	?
(*f*) New York . . .	71	82	?
(*g*) St. Louis	57	97	?
(*h*) Philadelphia . . .	55	98	?

§ 11. Marking Goods to Sell at a Profit on the Cost.

EXERCISES

Determine the marked price for each of the following:

	Cost	Per Cent of Profit on Cost	Profit	Marked Price
1.	\$1.12	25%	\$0.28	\$1.40
2.	2.70	$33\frac{1}{3}$%	?	?
3.	3.00	50%	?	?
4.	4.15	20%	?	?
5.	4.80	25%	?	?
6.	6.00	$37\frac{1}{2}$%	?	?
7.	7.50	50%	?	?
8.	12.60	25%	?	?
9.	37.80	25%	?	?
10.	52.00	$37\frac{1}{2}$%	?	?
11.	76.00	25%	?	?
12.	112.00	$37\frac{1}{2}$%	?	?
13.	162.50	40%	?	?
14.	175.00	30%	?	?
15.	212.00	60%	?	?
16.	260.00	55%	?	?
17.	310.00	48%	?	?
18.	324.00	$62\frac{1}{2}$%	?	?

II. Discounts

§ 12. Single Discounts. A *discount* is a reduction made from a given price. This reduction is generally stated as so many per cent of the given price. The given price is called the *list price*, the *marked price*, or the *catalogue price*. The price after the discount has been taken off is called the *net price*.

EXERCISES

Find the net price on each of the following at the discount quoted:

	Article	List Price	Discount	Net Price
1.	Saw	$3.50	10%	$3.15
2.	Shovel	1.75	20%	?
3.	Sled	4.50	15%	?
4.	Roller	8.50	10%	?
5.	Ash can	5.25	20%	?
6.	Lawn mower	9.75	$33\frac{1}{3}$%	?
7.	Wagon	57.50	15%	?
8.	Galvanized trough	18.25	20%	?

§ 13. Marking Down Goods.

EXERCISES

Find the selling price of each of the following articles:

	Article	Marked Price	Per Cent of Reduction	Selling Price
1.	Suit	$15.00	15%	$12.75
2.	Suit	12.50	10%	?
3.	Overcoat	16.50	20%	?
4.	Suit	20.50	15%	?
5.	Overcoat	25.00	15%	?
6.	Overcoat	30.00	25%	?
7.	Suit	35.00	$12\frac{1}{2}$%	?
8.	Fur coat	75.00	25%	?

§ 14. Trade Discounts.

Certain classes of goods such as jewelry, silverware, hardware, plumbing supplies, etc., are advertised by means of catalogues. When prices are quoted in such catalogues, they are subject to a series of discounts such as $33\frac{1}{3}\%$, 10%, and 2%. Such discounts are called *trade discounts*.

When a series of discounts are given, the first discount is reckoned on the catalogue price; the second discount on the price *after* the first reduction has been made; the third discount on the price *after* the second reduction has been made; etc.

EXAMPLE. A catalogue gives the price of a steam heater as \$160. This price is subject to a series of discounts of 50%, 10%, and 2%. Find the net price.

Solution.

	\$160	(Catalogue price)
Less $\frac{1}{2}$,	80	(Discount of 50% of \$160)
	80	
Less $\frac{1}{10}$,	8	(Discount of 10% of \$80)
	72	
Less 2%,	1.44	(Discount of 2% of \$72)
	\$70.56	(Net price)

Ans. \$70.56.

EXERCISES

Find the net price of each of the following articles:

	CATALOGUE	DISCOUNTS	NET PRICE
1.	\$210	$33\frac{1}{3}\%$, 10%	?
2.	240	$33\frac{1}{3}\%$, 10%	?
3.	270	$33\frac{1}{3}\%$, 10%	?
4.	300	$33\frac{1}{3}\%$, 10%	?
5.	460	25%, 5%	?
6.	840	25%, 5%	?
7.	1020	25%, 5%	?

	Catalogue	Discounts	Net Price
8.	$ 208	25%, 10%, 2%	?
9.	324	25%, 10%, 1%	?
10.	480	25%, 5%, 2%	?
11.	225	20%, 5%, 2%	?
12.	500	20%, 5%, 5%	?
13.	1605	20%, 2%	?
14.	775	10%, 5%	?
15.	850	20%, 5%, 2%	?

APPLIED PROBLEMS

1. A wholesale jewelry house gives a catalogue discount of 50% and a cash discount of 5%. Find the cost of each of the following articles, if cash is paid:

(*a*) An Elgin watch listed at $67.80.

(*b*) A Howard watch listed at $350.

(*c*) A Hamilton watch listed at $150.

(*d*) A bracelet watch listed at $38.70.

(*e*) A Waltham watch listed at $244.50.

(*f*) One dozen table spoons listed at $16.86.

(*g*) One dozen table forks listed at $18.42.

(*h*) One dozen table knives listed at $22.42.

2. Find the net cost of each of the following tires, discounts of 10% and 2% allowed:

(*a*) A tire listed at $36.35.

(*b*) A tire listed at $48.50.

(*c*) A tire listed at $16.10.

3. A heater manufacturing company quotes a catalogue discount of 50%, and an additional discount of 10% if cash accompanies the order. Find the net cost of each of the following:

(*a*) A hot-water heater listed at $460.

(*b*) A hot-water heater listed at \$535.

(*c*) A steam heater listed at \$480.

(*d*) A steam heater listed at \$285.

4. A lumber concern quotes a catalogue discount of 10%, and an additional discount of 2% for cash. If cash accompanies the order, find the net cost of each of the following:

(*a*) a carload of lumber listed at \$300.

(*b*) 6 screen doors at \$2.75 each.

(*c*) 225 bunches of cedar shingles at \$5.20 each.

(*d*) 25 doors at \$2.10 each.

(*e*) 25 door frames at \$1.80 each.

(*f*) 2500 sq. ft. galvanized roofing at \$3.70 per 100 sq. ft.

5. One dealer sells a certain tire, listed at \$32.50, at discounts of 10% and 5% while another dealer sells the same tire at a single discount of 15%. Which dealer offers the lower price? How much lower?

6. On a certain piano, listed at \$350, discounts of 15% and 5% are allowed by one dealer while discounts of 10% and 10% are offered on the same piano by a second dealer. Which dealer offers the lower price? How much lower?

7. Bathroom fixtures can be purchased of one dealer for \$200 less 25% and 10%. The same fixtures can be purchased from another dealer for \$180 less 20% and 5%. Which is the lower price? How much lower?

8. I can buy 500 suits of ready-made clothing at \$20 a suit from one firm less discounts of 10% and 10%. I can buy suits of the same grade from another firm at \$22.50 less discounts of 20% and 10%. Which is the lower price? What will be the difference in the total cost?

III. Interest

§ 15. Sixty-Day Method of Computing Interest.

A. In this method, the rate is taken as 6% for one year; then the rate for 60 days (2 mo.) is $\frac{1}{6}$ of 6% or 1%; and for 6 days it is $\frac{1}{10}$ of 1% or .1%.

Example. Compute the interest on \$240 for 3 months 8 days at 6%.

At 6%,

Int. on \$240 for 60 da. = \$2.40 (1% of \$240)
Int. on \$240 for 30 da. = 1.20 ($\frac{1}{2}$ of Int. for 60 da.)
Int. on \$240 for 6 da. = .24 (.1% of \$240)
Int. on \$240 for 2 da. = .08 ($\frac{1}{3}$ of Int. for 6 da.)
\$3.92

At 6%, the interest on \$240 for 3 mo. 8 da. is \$3.92.

Summary.

To find the interest for any given amount of time, express the total time in terms of 60 da., 6 da., and fractional parts thereof.

To find the interest for 60 da. (2 mo.), move the decimal point in the principal two places to the left (divide the principal by 100).

To find the interest for 6 da., move the decimal point in the principal three places to the left (divide the principal by 1000).

EXERCISES

At 6%, compute the interest on the following:

1. \$240 for 90 da.
2. \$560 for 4 mo. 18 da.
3. \$1240 for 75 da.
4. \$840 for 5 mo. 9 da.
5. \$760 for 7 mo. 15 da.
6. \$1380 for 105 da.
7. \$1595 for 10 mo. 10 da.
8. \$1685 for 8 mo. 5 da.
9. \$1390 for 1 yr. 3 mo. 6 da.
10. \$2380 for 9 mo. 14 da.

B. If the rate of interest is 5%, 4%, 4½%, or any per cent other than 6 per cent, we proceed as follows:

First, find the interest at 6%; then take such a fractional part of this answer as the given rate is of 6%.

EXAMPLE. Compute the interest on $840 for 5 months 9 days at 5%.

At 6%,

Int. on $840 for 60 da. = $8.40
Int. on $840 for 60 da. = 8.40
Int. on $840 for 30 da. = 4.20
Int. on $840 for 6 da. = .84
Int. on $840 for 3 da. = .42 (½ of Int. for 6 da.)

6)$22.26

(Since 5% is ⅚ of 6%, take ⅚ of $22.26.)

3.71
5
$18.55

or deduct ⅙

6)$22.26
3.71
$18.55

NOTE. Since 5 is also ⅙ less than 6, we may deduct ⅙ of the interest ($3.71) from $22.26. The result is the same.

Ans. At 5%, the interest on $840 for 5 mo. 9 da. is $18.55.

TABLE

$4\% = \frac{2}{3}$ of 6%	Deduct $\frac{1}{3}$ of interest at 6%
$4\frac{1}{2}\% = \frac{4.5}{6.0}$, or $\frac{3}{4}$ of 6%	Deduct $\frac{1}{4}$ of interest at 6%
$5\% = \frac{5}{6}$ of 6%	Deduct $\frac{1}{6}$ of interest at 6%
$5\frac{1}{2}\% = \frac{5.5}{6.0}$, or $\frac{11}{12}$ of 6%	Deduct $\frac{1}{12}$ of interest at 6%
$7\% = \frac{7}{6}$ of 6%	Add $\frac{1}{6}$ of interest at 6%
$8\% = \frac{4}{3}$ of 6%	Add $\frac{1}{3}$ of interest at 6%

EXERCISES

1. Find the interest in exercises 1–5, page 27, at 4%. (Deduct $\frac{1}{3}$.)

2. Find the interest in exercises 6–10, at 5%. (Deduct $\frac{1}{6}$.)

3. Find the interest in exercises 3–6 at $4\frac{1}{2}$%. (Deduct $\frac{1}{4}$.)

4. Find the interest in exercises 7–10 at $5\frac{1}{2}$%. (Deduct $\frac{1}{12}$.)

5. Find the amount due on \$3040 for 8 mo. 21 da. at $4\frac{1}{2}$%.

6. Find the amount due on \$4080 for 1 yr. 15 da. at 5%.

7. Find the amount due on \$3560 for 10 mo. 21 da. at 5%.

8. Find the amount due on \$15,000 for 9 mo. 15 da. at 4%.

9. Find the amount due on \$250,000 for 4 mo. 20 da. at $4\frac{1}{2}$%.

10. Find the interest due on \$2,000,000,000 (First Liberty Loan) for 6 mo. at $3\frac{1}{2}$%.

§ 16. Finding the Difference in Time between Two Dates.

A. Counting days.

EXAMPLE. How many days between Jan. 1 and Feb. 22?

30 days left in January
22 days in February
Total = 52 *Ans.* 52 days.

NOTE. Count the days left in January;
Count through the last date given, Feb. 22.

EXERCISES

Find the number of days between each of the following dates:

1. Sept. 16 and Oct. 12.
2. Jan. 15 and Feb. 12.
3. Oct. 12 and Nov. 8.
4. Feb. 22 and April 19.
5. Dec. 25 and Feb. 1. $(6 + 31 + 1.)$ Why?
6. April 19 and June 17.
7. April 19 and July 4.
8. July 4 and Sept. 1.

B. Counting by months and days.

EXAMPLE. Find the time between Sept. 5 and Dec. 25.
The time from Sept. 5 to Dec. 5 = 3 mo.
The time from Dec. 5 to Dec. 25 = 20 da.
Ans. 3 mo. 20 da.

NOTE. Count in months as far as possible; then count in days.

EXERCISES

Find in months and days the time between each of the following dates:

1. Sept. 1 and Nov. 29.
2. July 4 and Jan. 1.
3. Jan. 1 and June 17.
4. April 19 and Oct. 12.
5. April 1 and Oct. 28.

C. Reading from the table.

To find the difference in time between two dates, the following is a form of table used by bankers:

From — to	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any month within a year is found opposite the first month and under the second.

Example 1. How many days are there from April 6 to July 6?

Find April in the column at the left; then in the same line under July (at the top) you find 91. Hence, there are 91 days from April 6 to July 6. *Ans.* 91 da.

Example 2. How many days from April 6 to July 25?

To the 91 days just found from the table (in example 1), add 19 days more, since there are 19 days from July 6 to July 25. Hence, there are 110 days from April 6 to July 25. *Ans.* 110 da.

EXERCISES

Find the number of days between each of the following dates:

1. July 4 to Sept. 15.
2. Aug. 9 to Nov. 20.
3. March 3 to July 4.
4. Jan. 5 to April 19 (leap year).

Note. In a leap year add one day if Feb. is included.

5. Dec. 2 to Feb. 9.
6. Sept. 17 to Jan. 1.
7. Oct. 28 to April 1 (leap year).
8. April 5 to Aug. 3.

D. Application to interest.

EXERCISES

Find the interest in the following exercises:

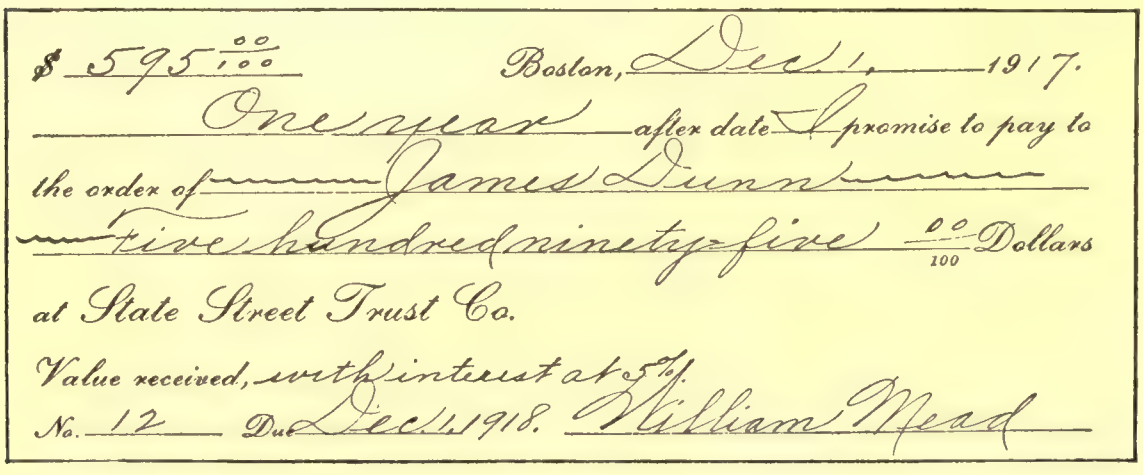
Use method (*A*) in Exs. 1–4; method (*B*) in Exs. 5–8; method (*C*) in Exs. 5–8.

1. $184 from Sept. 1 to Oct. 12, at 5%.
2. $460 from Oct. 15 to Dec. 1, at 5%.
3. $845 from Sept. 12 to Nov. 1, at 4%.
4. $1080 from July 1 to Aug. 8, at 4%.
5. $375 from Jan. 1 to April 15, at 5%.
6. $970 from March 15 to June 30, at 4%.
7. $1560 from April 1 to Sept. 6, at 5%.
8. $3040 from Sept. 6 to Jan. 1, at 5%.

§ **17. Notes.** A ***note*** is a written promise to pay money.

Money loaned at interest is usually secured by a note.

The form and wording of a note is usually about as follows:


$595.00 Boston, Dec. 1, 1917.
One year after date I promise to pay to the order of James Dunn
Five hundred ninety-five 00/100 Dollars
at State Street Trust Co.
Value received, with interest at 5%.
No. 12 Due Dec. 1 1918. William Mead

The main points to be noticed are:

(*a*) The amount of the note written in figures at upper left-hand corner; in words in body of note.

(*b*) The place and date.

(*c*) The rate of interest.

(*d*) The name of the *payee*, James Dunn, the man to be paid.

(*e*) The signature of the *maker*, William Mead, the man to pay the note.

EXERCISES

1. Compute the amount due on the above note, if it is paid at maturity (the date when it is due).

2. On Jan. 5, 1917, Mr. J. Minch buys a horse from Mr. A. Levy for $285. He pays one hundred dollars in cash and gives his note, due in 6 mo., interest at 5%, for the balance. Write this note. Compute the interest, and the amount, due at maturity.

3. The International Harvester Company sells on July 1 to Layton Smith one reaper for $85. Mr. Smith gives his note for 9 mo., interest at 5%, in payment. Write this note. Compute the amount due at maturity.

4. Mr. Henry Maxon buys an automobile of Mr. John Corson for $595. He pays $300 in cash (when the car is delivered) on April 15, 1917. He gives his note, payable one year after date, interest at 6%, for the balance. Write this note. Compute the amount due Mr. Corson on April 15, 1918.

5. A Coöperative Bank holds a note for $2450 against *A* at $5\frac{1}{2}\%$ interest. The bank requires *A* to pay this interest monthly in advance. How much interest does *A* have to pay on this note each month?

§ 18. The Interest Formula.

In computing simple interest, you have observed that the interest is found by multiplying the principal by the rate per cent, and this product by the time expressed in years, or fractional parts of a year.

This statement,

$$interest = principal \times rate \times time \text{ (in } years\text{)},$$

may be briefly expressed by the formula,

$$i = prt,$$

where p = the *principal*,
r = the *rate* (per year),
t = the *time* (in years),
i = the *interest*.

This is known as the *interest formula*.

EXAMPLE 1. Compute the interest on \$295 at 5% for 1 yr. 3 mo.

SOLUTION. $i = prt$

$$i = 295 \times \frac{5}{100} \times \frac{15}{12},$$

$$i = 295 \times \frac{\cancel{5}}{\cancel{100}\ \cancel{20}\ 4} \times \frac{\overset{3}{\cancel{15}}}{\cancel{12}\ 4} = \frac{295}{16},$$

$$i = 18.437$$

Ans. The interest = \$18.44.

EXAMPLE 2. The interest at $4\frac{1}{2}\%$ on a given principal for 8 mo. 10 da. is \$25. Find the principal.

SOLUTION. $i = prt$

$$25 = p \times \frac{\overset{5}{\cancel{45}}}{\cancel{1000}\ \cancel{40}\ 8} \times \frac{\cancel{250}}{\cancel{360}\ 4},$$

$$25 = \frac{p}{32}.$$

Multiplying each member by 32,

$800 = p.$ *Ans.* The principal = \$800.

EXAMPLE 3. In what time will \$650 at 5% interest produce \$48.75?

SOLUTION. $i = prt$

$$\$48.75 = 65\cancel{0} \times t \times \frac{\cancel{5}}{\cancel{100}\,2},$$

Multiplying each member by 2,

$97.50 = 65 \times t.$

Dividing each member by 65,

$1.5 = t.$ *Ans.* The time = 1.5 yr.

EXERCISES

Find the required number in each of the following exercises:

FORMULA: $i = prt$.

	INTEREST	PRINCIPAL	TIME	RATE
1.	?	\$1080	9 mo. 15 da.	$5\frac{1}{2}\%$
2.	?	2560	1 yr. 5 mo.	4%
3.	\$7.80	?	9 mo.	4%
4.	67.50	?	1 yr. 6 mo.	$4\frac{1}{2}\%$
5.	18.75	?	8 mo. 12 da.	5%
6.	30.40	760	?	6%
7.	83.20	1560	?	4%
8.	500.00	?	1 yr.	5%
9.	1200.00	?	1 yr.	4%
10.	30.00	?	3 mo.	$3\frac{1}{2}\%$
11.	500.00	500	?	4%
12.	1000.00	1000	?	$4\frac{1}{2}\%$

§ 19. Savings Banks. Money deposited in a savings bank draws interest at a certain rate per cent per year. This interest, if left in the bank, is added to the principal, usually twice a year, on stated dates.

Interest that is added to the principal in this way is called ***compound interest***. In some banks interest begins to accumulate from the first day of the calendar month on all sums on deposit at that time.

In computing the interest due on a given date, banks usually observe the following rules:

(*a*) Interest is paid on the dollars of the principal only, — not on the fractional parts of a dollar.

(*b*) Interest is reckoned from the first day of each calendar month on the amount on deposit on that date.

EXAMPLE. Below is a sample page of a savings bank book. The rate of interest paid was 4%; the dates of compounding the interest were Jan. 1 and July 1.

DATE	DEPOSIT	INTEREST	WITHDRAWAL	BALANCE
March 1	$50			$50
July 1	50	67		100 67
Jan. 1		2 00		102 67

NOTE. The *balance* recorded on any date includes the deposit made on that date.

The interest on $50 from March 1 to July 1 (4 mo.) at 4% was $.67.

The amount on deposit on July 1, including the deposit made on that date, was then $100.67.

The interest on $100 (dollars only) from July 1 to Jan. 1 at 4% was $2.00. The balance then was $102.67.

Ans. $102.67.

EXERCISES

In each exercise that follows, compute the amount of the balance on the last date given. (Dates of compounding are Jan. 1 and July 1.)

1. RATE OF INTEREST, 4%

DATE	DEPOSIT	INTEREST	WITHDRAWAL	BALANCE
Sept. 1 . .	$25			$25
Oct. 29 . .	40			65
Jan. 1 . .		?		?

NOTE. Include the interest on $25 for 4 mo. + interest on $40 for 2 mo.

2. RATE OF INTEREST, 3%

Aug. 30 . .	$120			$120
Nov. 1 . .	60			180
Jan. 1 . .		?		?

3. RATE OF INTEREST, $3\frac{1}{2}$%

Feb. 21 . .	$175			$175
May 28 . .	60			235
July 1 . .		?		?
Jan. 1 . .		?		?

4. RATE OF INTEREST, 3%

June 25 . .	$225			$225
Oct. 28 . .	25			250
Jan. 1 . .	50	?		?

IV. The Percentage Formula

§ 20. Finding the Base.

The statement, *percentage* = *rate* × *base*, may be briefly expressed by the formula,

$$p = r \times b,$$

where

b = the *base* (the number on which the percentage is found),

r = the *rate* (the number of hundredths to be taken),

p = the *percentage* (the number found by taking a certain per cent of the base).

When the base in a given example is not known, the percentage formula gives a simple method of finding it.

Example 1. 12 is 50% of what number?

Solution. Let b (base) = the number, then,

$$12 = .50\, b,$$

$$12 = \tfrac{1}{2}\, b.$$

Multiplying each member by 2,

$$24 = b.$$

Check. *In the statement of the given example,* substituting 24 for the number,

$$12 \stackrel{?}{=} 50\% \text{ of } 24,$$

$$12 = 12. \quad \textit{Ans.}\ 12 = 50\% \text{ of } 24.$$

Example 2. 36 is 75% of what number?

Solution. Let b (base) = the number, then,

$$36 = .75\, b,$$

$$36 = \tfrac{3}{4}\, b.$$

Multiplying each member by 4,

$$144 = 3\, b.$$

Dividing each member by 3,

$$48 = b.$$

CHECK. Substituting 48 for the number,

$$36 \stackrel{?}{=} 75\% \text{ of } 48,$$

$$36 = 36. \quad \textit{Ans.}\ 36 = 75\% \text{ of } 48.$$

EXAMPLE 3. 84 is 14% of what number?

SOLUTION. Let b (base) = the number, then,

$$84 = .14\ b.$$

Multiplying each member by 100,

$$8400 = 14\ b.$$

Dividing each member by 14,

$$600 = b.$$

CHECK. Substituting 600 for the number,

$$84 \stackrel{?}{=} 14\% \text{ of } 600,$$

$$84 = 84. \quad \textit{Ans.}\ 84 \text{ is } 14\% \text{ of } 600.$$

EXAMPLE 4. In a certain school there are 348 girls; 58 per cent of all the pupils are girls. How many pupils are there in this school?

NOTE. Here the base is the total number of pupils in this school.

SOLUTION. Let b (base) = the number of pupils, then,

$$.58\ b = \text{the number of girls},$$

$$.58\ b = 348.$$

Multiplying each member by 100,

$$58\ b = 34800.$$

Dividing each member by 58,

$$b = 600.$$

CHECK. Substituting 600 for number of pupils in school,

$$348 \stackrel{?}{=} 58\% \text{ of } 600,$$

$$348 = 348.$$

Ans. There are 600 pupils in school.

EXAMPLE 5. In a certain school there were 540 pupils registered in 1916. This number is 8 per cent more than the number of pupils registered in 1915. How many pupils were registered in 1915?

SOLUTION. Let b = number of pupils in 1915, then,

108% of b = number of pupils in 1916,

$1.08\ b$ = number of pupils in 1916,

$1.08\ b = 540.$

Multiplying each member by 100,

$$108\ b = 54000.$$

Dividing each member by 108,

$$b = 500.$$

CHECK. Substituting 500 for the number registered in 1915,

$540 \stackrel{?}{=} 8\%$ more than 500,

$540 = 540.$

Ans. There were 500 pupils in 1915.

EXAMPLE 6. In a certain school there were 690 pupils registered in 1916. This number is 8 per cent less than the number of pupils registered in 1915. How many were registered in 1915?

SOLUTION. Let b = the number in 1915, then,

92% of b = the number in 1916,

$.92\ b$ = the number in 1916,

$.92\ b = 690.$

Multiplying each member by 100,

$$92\ b = 69000.$$

Dividing each member by 92,

$$b = 750.$$

Ans. There were 750 pupils in 1915.

Let the student check this answer.

EXERCISES

In each of the following exercises, find the *base:*

FORMULA: $p = rb$.

	PERCENTAGE	RATE	BASE
1.	\$1000	$62\frac{1}{2}\%$	?
2.	\$960	12%	?
3.	\$540	$37\frac{1}{2}\%$	?
4.	1728 in.	16%	?
5.	2150.42 cu. in.	95%	?
6.	\$21.50	2%	?
7.	1760 yd.	$83\frac{1}{3}\%$	?
8.	5280 ft.	125%	?

APPLIED PROBLEMS

1. In a certain school 42% of the pupils are boys; the number of boys in this school is 441. Find the total number of pupils in this school.

2. In a certain school 390 pupils have defective vision; this number is 52% of all the pupils in the school. How many pupils are there in this school?

3. In a certain school 15% of the pupils failed of promotion; 87 pupils were not promoted. Find the total number of pupils in this school.

4. A dealer sold a horse for \$250; this was 25% more than the cost. What was the cost of the horse?

5. A merchant's profits were found to be 28% of the capital invested; his profits were \$2800. How much did he have invested?

6. The cost of a bushel of potatoes on April 1, 1917, was \$3.40; this was 70% more than the cost of a bushel on April 1, 1916. What was the cost on April 1, 1916?

7. The cost of a bag of flour on April 1, 1917, was \$1.53; this was 62% more than the cost of the same grade of flour in 1916. What was the cost on April 1, 1916?

8. A man's salary in 1917 was \$1260. This was $12\frac{1}{2}\%$ more than it was in 1916. What was his salary in 1916?

9. In a lumber mill the waste in sawing odd lengths was estimated to be 2.3% of the number of board feet sawed. In one day the number of board feet wasted in a certain mill was 430. What was the total number of board feet of lumber sawed that day?

10. In a second mill the number of board feet wasted in one day was 359. What was the total number of board feet of lumber sawed in this mill on this day?

11. Charles sold his bicycle, after using it for one year, for \$19.50; this was 25% below the price he paid for it. What did he pay for it?

12. After using an automobile for one season I sold it for \$289; this was 15% less than what I paid for it. What did I pay for it?

13. A dealer sold a damaged desk for \$22.50; this was 10% below what the desk cost him. What did the desk cost him?

14. A real estate agent sold two houses at \$6000 each. On the one he gained 10% of the cost, on the other he lost 10% of the cost. Find his net gain or loss on this transaction.

15. A dealer sold two horses at \$240 each. On the one he gained 25% of the cost, on the other he lost 25% of the cost. Did he gain or lose on this transaction? How much?

§ 21. Profit Reckoned as a Per Cent of the Selling Price.

It is now customary for many retail, and some wholesale, houses to reckon profit as a per cent of the selling price, rather than as a per cent of the cost.

A few of the more important reasons advanced by them for this practice are:

(*a*) Commissions to agents are based on their total sales.

(*b*) Taxes are based on the gross sales.

(*c*) The expense of conducting business is based on gross sales.

(*d*) No profit is realized until a sale is effected.

In any problem dealing with a per cent of profit, it is necessary to know on what *base* the profit is reckoned; that is, whether it is on the cost or on the selling price.

The percentage formula will give a simple method for determining the selling price, when a given percentage of profit on the selling price is desired. Here the selling price (sometimes called marked price) is the *base*, the cost is the *percentage*, and the per cent of profit is the *rate*.

EXAMPLE. At what price should a merchant sell a coat costing \$9, so as to gain 25 per cent on the selling price?

SOLUTION.

$p = r \times b$ becomes c (cost) $= r \times s$ (selling price),

then 25% of s = the profit,

and 75% of s = the cost,

hence $.75\ s = 9$.

Multiplying each member by 100,

$$75\ s = 900.$$

Dividing each member by 75,

$$s = 12.$$

CHECK. Substituting \$12 for the selling price,

$$75\% \text{ of } 12 \stackrel{?}{=} 9,$$

$$9 = 9.$$

Ans. The selling price is \$12.

EXERCISES

1. The following articles are to be sold at the given per cent of profit on the selling price. Determine the selling price for each article listed below.

	ARTICLE	COST	PROFIT ON SELLING PRICE	SELLING PRICE
(*a*)	Hammer	\$0.75	40%	?
(*b*)	Saw	1.20	$33\frac{1}{3}\%$	?
(*c*)	Square	.90	50%	?
(*d*)	Plane	1.50	25%	?
(*e*)	Level	2.20	40%	?
(*f*)	Steel tape	5.50	$33\frac{1}{3}\%$	?
(*g*)	Hat	1.30	60%	?
(*h*)	Shoes	2.25	75%	?
(*i*)	Gloves	.50	60%	?
(*j*)	Suit	8.50	50%	?
(*k*)	Suit	10.50	60%	?
(*l*)	Suit	12.00	40%	?
(*m*)	Overcoat	14.00	$33\frac{1}{3}\%$	?

2. The following articles are to be marked to sell at a profit of 30% on the selling price. Determine the selling price for each: (*a*) \$42. (*b*) \$56. (*c*) \$73.50. (*d*) \$126.

NOTE. When the percentage of profit is computed on the selling price, there can never be a profit of 100%, unless the cost of the goods is nothing, which of course is absurd.

3. A suit, which costs \$8, is sold for \$14.
(*a*) What is the profit?
(*b*) The profit is what per cent of the cost?
(*c*) The profit is what per cent of the selling price?

4. A suit, which costs \$15, is sold for \$30.
(*a*) What is the profit?
(*b*) The profit is what per cent of the cost?
(*c*) The profit is what per cent of the selling price?

5. An article costing \$37.50 is sold for \$112.50.
(*a*) What is the profit?
(*b*) The profit is what per cent of the cost?
(*c*) The profit is what per cent of the selling price?

6. A pair of skates costing \$3.25 is sold for \$5.
(*a*) What is the profit?
(*b*) The profit is what per cent of the cost?
(*c*) The profit is what per cent of the selling price?

The following table compares the percentage of profit when determined on the cost with the percentage of profit when determined on the selling price:

Profit on Cost	Profit on Selling Price
10%	$9\frac{1}{11}\%$
$11\frac{1}{9}\%$	10%
20%	$16\frac{2}{3}\%$
25%	20%
$33\frac{1}{3}\%$	25%
50%	$33\frac{1}{3}\%$
$66\frac{2}{3}\%$	40%
75%	$42\frac{6}{7}\%$
100%	50%
150%	60%
300%	75%
900%	90%

MISCELLANEOUS PROBLEMS IN PERCENTAGE

1. The price of a bushel of wheat in the United States on Nov. 1, 1915, was 93 cents; on Nov. 1, 1916, it was 60% higher. What was the price in 1916?

2. A table marked to sell at \$65 was discounted $12\frac{1}{2}\%$. At what price was it sold?

3. On an order of books a teacher obtained a discount of $16\frac{2}{3}\%$. If the order amounted to \$37.50, how much was paid for the books?

4. A baseball team wins $62\frac{1}{2}\%$ of its games. It has played 24 games. How many games has it lost?

5. Of a school of 780 pupils, 763 were present on a certain day. What was the per cent of attendance?

6. Flour is selling at \$1.50 a bag, or \$9.90 per barrel. There are 8 bags in a barrel. What per cent is saved by buying a barrel?

7. Coal sells at \$8.00 a ton (2000 lb.), or at 50 ¢ per bag of 100 lb. What per cent is saved by buying a ton?

8. Potatoes are selling at 49 cents a peck, or at \$1.60 per bushel. What per cent is saved by buying a bushel?

9. The price of a barrel of apples in 1915 was \$2.10; in 1916, \$2.60. What was the per cent of increase?

10. A pair of shoes that sold for \$5.00 in 1915 sold for \$6.25 in 1916. What was the per cent of increase?

11. A merchant marks his goods to sell at a gain of 25% on the selling price. At what price should he mark articles costing as follows: Table, \$25.50? Chair, \$7.50? Sideboard, \$30.90?

12. The window area of a school room should be 25% of the floor area. If the floor area is 816 sq. ft., what should be the window area?

13. In a school of 2216 pupils, $12\frac{1}{2}\%$ fail of promotion. How many fail? How many are promoted?

14. In a school of 1620 pupils, 425 fail of promotion. What per cent are promoted?

15. On a piano listed at \$775 discounts of 20% and 2% are given. Find the net cost.

16. The price of a railroad ticket is raised from \$2.10 to \$2.40. What is the per cent of increase?

17. The number of trains stopping at a certain station is reduced from 80 to 72. Find the per cent of reduction.

18. The pay of a carpenter is raised from \$5.10 per day to \$5.25. What is the per cent of increase?

19. The number of passengers traveling on a certain car line in the city increased per day from 3250 to 5000. What was the per cent of increase?

20. The average pay of a wage earner in a city is increased from \$484 to \$518 per year. What is the per cent of increase?

21. The floor area of an assembly hall in a school is 36,000 sq. ft. The area of a class room is 450 sq. ft. The floor area of the class room is what per cent of that of the assembly hall?

22. The area of a desk cover in a class room is $3\frac{3}{4}$ sq. ft. There are 42 desks in the room. If the room is $17\frac{1}{2}$ ft. by 22 ft., what per cent of the floor area is the total area of the desk covers?

23. A window pane in the class room (Ex. 22) is 14 in. by 9 in. There are four windows, each with 18 panes of glass. What per cent of the floor area is the window area?

24. In a certain school there are 10,560 books and 350 pupils. Each pupil draws on an average 2 books per week. What per cent of the books are in use each week?

CHAPTER III

ARITHMETIC OF THE HOME

I. Personal Accounts

§ 22. Personal Accounts.

The economic success of an individual depends upon three things: (1) his earning power, (2) his wisdom in spending, (3) his habit of saving.

You have heard much about earning and saving money, but very little about the importance of spending it wisely.

In order that you may spend wisely and carefully you should keep a personal cash account. In this account you should keep a record of all money received, telling from what source, — such as, allowance, gifts, earnings, etc. Also a record of all money paid out, telling for what purpose, — such as, clothes, lunches, car fare, etc.

A simple form to use in keeping personal accounts is the two-column cash book (page 49).

The first column is used for money received, the second column for money paid out. Every item should be dated and explained.

The cash account should be balanced often, at least weekly; also at the bottom of each page.

To balance the cash account: Add each side of the cash book and find the difference. Write the difference as *Balance* in the *Paid* column. Rule it as illustrated and bring the *Balance* down in the *Received* column for the new account. [The balance is usually written in red ink.]

The following form illustrates the method of making entries, balancing and ruling:

PERSONAL CASH BOOK

		January 1–8, 1917	Received	Paid
Jan.	1	Received allowance	$1 00	
	2	Paid for lunch		12
	3	Paid for notebook		10
	4	Paid for candy		10
	5	Paid for entertainment		25
	5	Paid for car fare		30
	6	Paid for lunch		15
	6	Deposited in School Savings Bank		20
	7	Received for shoveling snow	25	
	7	Received for doing errands	10	
		Balance *		13
			1 35	1 35
Jan.	8	Balance on hand	13	
	8	Received allowance	1 00	

EXERCISES

Rule a sheet of paper as shown in the illustration and enter the following transactions in it.

1. Sept. 1. Received allowance, $2.00.
1. Paid for cap, 35¢.
3. Paid for soda, 10¢.
4. Paid for tennis ball, 25¢.
6. Paid for Gym. shoes, 75¢.
7. Paid for Church and Sunday School, 7¢.
8. Deposited in Bank, 25¢.

Balance the cash book.

2. Sept. 8. Balance on hand, ? [Use balance in Ex. 1.]
8. Received allowance, $2.00.
9. Paid for trip to country, 78¢.

* *Balance* is the excess of the total amount received over the total amount paid out.

10. Paid for stockings, 25¢.
11. Paid for soda, 10¢.
12. Paid for movies, 10¢.
13. Paid for writing paper, 35¢.
14. Paid for Church and Sunday School, 7¢.
15. Deposited in Bank, 25¢.

Balance the cash book.

3. Sept. 15. Balance on hand, ? [Ex. 2.]
15. Received allowance, $2.00.
16. Paid for 2 notebooks, 25¢.
17. Paid for 2 pencils, 10¢.
18. Paid for blouse, $1.00.
20. Paid for football game, 25¢.
20. Paid for sodas, 20¢.
21. Paid for Church and Sunday School, 7¢.
22. Deposited in Bank, 25¢.

Balance the cash book.

4. Sept. 22. Balance on hand, ? [Ex. 3.]
22. Received allowance, $2.00.
23. Paid for sweater, $1.25.
24. Received for work, 25¢.
24. Paid for underclothes, 45¢.
27. Paid for football game, 25¢.
28. Paid for Church and Sunday School, 7¢.
29. Deposited in Bank, 20¢.

Balance the cash book.

5. Rule a sheet of paper as shown in the illustration and enter the following transactions in it, and show the balance after each week (Feb. 7, Feb. 14, etc.).

Feb. 1. Received allowance, $1.25.
2. Paid for handkerchiefs, 25¢.

3. Paid for car fare, 20¢.
4. Paid for church, 10¢.
5. Paid for magazine, 15¢.
6. Received for household duties, 25¢.
7. Paid for notebook, 10¢.
8. Received allowance, $1.25.
10. Paid for sharpening skates, 25¢.
11. Paid for soda and candy, 15¢.
13. Paid for car fare, 25¢.
14. Paid for concert, 50¢.
15. Received allowance, $1.25.
16. Paid for Church and Sunday School, 15¢.
18. Received for doing errands, 15¢.
20. Paid for rubbers, 60¢.
21. Paid for book, 50¢.
22. Received allowance, $1.25.
24. Paid car fare, 25¢.
26. Paid for notebook, 20¢.
28. Paid for gloves, 75¢.

When you want to know at the end of the month how much has been spent for *each* purpose, it is necessary to draw off a list as follows:

SUMMARY OF EXPENSES

MONTH	TOTAL PAYMENTS	CLOTHING	CAR FARE	AMUSEMENT	EDUCATION	CHURCH	MISCELLANEOUS	SAVINGS
Sept.	8 01	4 05	78	85	70	28	40	95

FAMILY ACCOUNTS

DATE	RECEIPTS	TOTAL PAYMENTS	RENT	FOOD				OPERATING EXPENSES				CLOTHING	HIGHER LIFE					
				Milk and Butter	Meat	Groceries	Fruit and Vegetables	Coal	Gas and Light	Laundry	Miscellaneous		Insurance	Recreation	Papers and Magazines	Church and Charity	Health	Savings
Jan.	80 00	75 55	16 00	3 25	7 50	12 00	2 00	4 50	1 75	1 25	3 50	12 00	3 00	1 70	80	80	1 50	4 00
Feb.	4 45																	
Mar.	80 00	77 55	16 00	3 15	7 25	12 50	2 10	4 50	1 65	1 20	3 70	13 50	3 00	1 50	75	75	2 00	4 00
	6 90																	

II. Family Accounts and Budgets

§ 23. Family Accounts. In order that habits of thrift may be formed, every family should keep a record of the expenditures for each month. Herewith is shown a simple form of keeping the family account in which the expenditures for various purposes are classified.

EXERCISES

1. A family of three has an income of $90 a month; their expenses for the month of February are as follows:

Rent, $18.00	Laundry, $.30	Groceries, $13.18
Meat, $8.10	Concert, $.50	Vegetables, $1.19
Coal, $5.00	Church, $.45	Rubbers, $.75
Gas, $1.60	Doctor, $.75	Coöperative Bank, $4.00
Fruit, $1.26	Shoes, $4.00	Tooth-paste, $.20
Milk, $2.40	Laundry, $.35	Newspaper, $.30
Shoes, $3.00	Butter, $1.20	Sweater, $3.00
Hat, $2.00	Theater, $.50	Suit of clothes, $16.00

Rule a sheet of paper as shown in the illustration and enter the various items in the proper columns. Find the total expenditures and the balance.

2. A family of five has an income of $120 a month; their expenses for the month of April are as follows:

Rent, $22.00	Butter, $2.23
Groceries, $15.62	Savings, $5.00
Coal, $7.50	Overcoat, $12.00
Insurance, $10.80	Fruit, $2.74
Entertainment, $1.50	Vegetables, $2.15
Laundry, $.75	Hat, $2.00
Meat, $6.25	Gloves, $1.50
Gas and light, $2.50	Shoes, $3.00
Milk, $2.10	Magazines, $.50
Church, $1.75	Clothing, $7.65
Theater, $1.00	Miscellaneous, $8.00

As in Ex. 1, find the total expenditures and the balance. What per cent of the income was spent for rent? For food? For clothing? (Find each to the nearest per cent.)

3. A family of four has an income of $135 a month; their expenses for the month of June are as follows:

Rent, $30.00
Fuel, $3.25
Savings, $13.50
Groceries, $16.12
Entertainment, $3.50
Magazine (1 year), $3.00
Church and charity, $5.00
Meat, $8.10
Fruit and vegetables, $4.50
Service, $4.20
Gas and light, $1.80
Clothes, $26.80
Milk and butter, $3.20
Health, $3.10
Paper, $.55
Laundry, $2.20

As in Ex. 1, find the total expenditures and the balance. What per cent of the income was spent for rent? For food? For clothing? (Find each to the nearest per cent.)

4. A family of six has an income of $150 a month; their expenses for the year are as follows:

Rent, $336.00
Coal, $64.00
Gas and light, $32.50
Insurance:
  Fire, $8.40
  Life, $110.00
Savings, $150.00
Books, $12.00
Water, $15.00
Health, $50.00
Service, $22.50
Postage and express, $8.60
Vacation trip, $50.00
Church and charity, $56.60
Clothing, $252.50
Magazines, $5.50
Papers, $7.20
Food, $322.40
Miscellaneous, $112.50
Furniture, $37.50
Entertainments, $28.50

As in Ex. 1, find the total expenditures and the balance. What per cent of the income was spent for rent? For food? For clothing? (Find each to the nearest per cent.)

§ 24. Family Budgets. In order that the family income may be used to the best advantage, it is necessary to plan in advance the divisions of the income for the necessary purposes. Such a plan is called a ***family budget.***

Investigations in this country show that in a family of four, consisting of two adults and two children under 16 years of age, the expenditures should be divided as follows. Conditions will vary in different sections of the country, and with the community. Hence, this table expresses only average conditions.

Income	Food	Rent	Operating Expenses	Clothing	Higher Life
Ideal . . .	25%	20%	15%	15%	25%
$1500–$4000 . . .	25%	20%	15%	20%	20%
$1000–$1500 . . .	30%	20%	15%	20%	15%
$800–$1000 . . .	35%	20%	15%	15%	15%
$500–$800 . . .	45%	15%	15%	15%	10%
Under $500 . . .	60%	15%	10%	10%	5%

EXERCISES

1. A family, consisting of two adults and two children under 16 years of age, has an income of $700. How much, according to the above table, may be expended for each purpose?

Note. The income is between $500 and $800.

2. How much may the family in Ex. 1 spend for each purpose, when their income has been increased to $900 a year?

When it has been increased to $1100? When it has been increased to $2000?

3. The members of a certain family are Mr. and Mrs. Jones; a son, James, aged 15 years; a daughter, Mary, aged 10 years. Mr. Jones earns \$3.75 a day for an average of 300 days in the year; James earns \$1.50 per week selling papers and doing errands, for an average of 50 weeks in the year.

(*a*) What is their total income for the year?

(*b*) How much may the Jones family pay for food for a year? For rent? For clothing?

(*c*) What is the average amount to be paid per month for food?

(*d*) Allowing 12% for milk and butter, 30% for meat, 58% for all other groceries, how much can be expended for each?

(*e*) What amount is to be paid per month for rent?

(*f*) What amount is to be paid per month for operating expenses?

(*g*) Allowing 35% for coal, 20% for gas and light, how much will be left for miscellaneous operating expenses?

(*h*) What amount is to be paid for clothing for the year?

(*i*) Allowing Mrs. Jones 40%, Mr. Jones 30%, James 15%, Mary 15%, how much may each spend for clothing for the year?

(*j*) What amount is to be paid per month for higher life?

(*k*) Allowing 25% for life insurance, $6\frac{2}{3}$% for church and benevolence, 25% for recreation, books and magazines, $33\frac{1}{3}$% for savings, 10% for health, how much should be expended for each?

FOOD PROBLEMS

1. A package of crackers containing 10 oz. sells for 10¢; the same kind of crackers are 10¢ per pound in bulk. What per cent is saved on a pound by buying crackers in bulk?

2. A package containing 44 biscuits, net weight $6\frac{1}{2}$ oz., sells for 10¢. How many biscuits in a pound? How much would a pound cost?

3. A package of crackers containing $5\frac{1}{4}$ oz. sells for 10¢. How much is this per pound?

4. A can of fish which contains 13 oz. sells for 40¢. How much does the fish cost per pound?

5. A package of pancake flour which contains 1 lb. 4 oz. (net weight) sells for 10¢. How much is it per pound? How much cheaper per pound would it be to use a mixture of flour costing $1.50 per bag, if the bag contains $24\frac{1}{2}$ lb.?

6. What fractional part of a barrel of flour is a bag of flour weighing $24\frac{1}{2}$ lb.? What per cent? (1 bbl. flour weighs 196 lb.)

7. How much is saved by purchasing a barrel of flour for $12.00, rather than by paying $1.65 per bag of $24\frac{1}{2}$ lb.?

8. When apples sell for $1.00 a bushel, what will be the cost of one peck? Of 4 quarts? How much should one peck weigh? 4 quarts? (1 bu. apples weighs 48 lb.)

9. When 5 lb. of beans cost 60 cents, what is the cost of a bushel? Of a quart (to the nearest cent)? (1 bu. beans weighs 60 lb.)

10. If you buy 5 lb. of tomatoes, how many quarts will you receive? (1 bu. tomatoes weighs 56 lb.)

III. Bills and Checks

§ 25. Bills. A ***bill*** is a statement of money due.

Example 1.

Newtonville, Mass., Dec. 11, 1916.

Mr. Henry Meade

Bought of Henry W. Bates
285 Walnut Street

1916				
Nov. 5	2 lb. butter at	42¢	$	84
7	$2\frac{3}{4}$ lb. steak at	48¢	1	32
12	2 pk. potatoes at	54¢	1	08
12	$\frac{1}{4}$ lb. cream of tartar at	60¢		15
			$3	39

Received payment,
Dec. 14, 1916. Henry W. Bates.

The main points to be noticed are:

(*a*) Place and date are given.

(*b*) Name of purchaser, Henry Meade, called the *debtor*.

(*c*) Name of seller, Henry W. Bates, called the *creditor*.

(*d*) Date of each purchase.

(*e*) Number and price of each article.

(*f*) Cost of each, and total cost of all articles.

(*g*) Form of receipting the bill.

Example 2.

Newton, Mass., Sept. 1, 1916.

Mr. James Todd,
To Patrick Hart, Dr.

July 8	3 hr. work	at 33¢	$	99
14	$2\frac{1}{2}$ hr. work	at 33¢		83
31	4 hr. work	at 33¢	1	32
Aug. 15	$5\frac{1}{2}$ hr. work	at 33¢	1	82
			$4	96

Received payment,
Patrick Hart.

Mr. James Todd is the man to pay the bill. Mr. Patrick Hart is the man to be paid.

EXERCISES

1. Harold Brown is hired by Mr. Smith to shovel his walks at 15¢ per hour. In January, Harold works as follows: Jan. 5, 3 hr.; Jan. 6, 2 hr.; Jan. 12, 3 hr.; Jan. 17, 4 hr.; Jan. 25, $2\frac{1}{2}$ hr. Make out as in Example 2 the bill which Harold sends to Mr. Smith on Feb. 1.

2. Mr. Rider orders from A. C. Fletcher the following: 1 clothes reel at \$2.75; 6 lb. staples at 5¢; 1 rake at 45¢; 5 lb. lawn seed at 25¢; $11\frac{1}{2}$ ft. wire at 5¢. Make out the bill to-day and find the amount of it.

Make out bills for the following, using your name as the buyer and the name of some firm (or storekeeper) for the seller.

3. $2\frac{1}{2}$ lb. butter at 42¢.
$1\frac{1}{4}$ lb. cheese at 22¢.
$1\frac{1}{2}$ lb. lard at 14¢.
$\frac{1}{2}$ lb. tea at 46¢.

4. $\frac{1}{2}$ doz. eggs at 52¢.
$\frac{1}{2}$ doz. oranges at 25¢.
$1\frac{1}{4}$ lb. pork chops at 30¢.
$1\frac{3}{4}$ lb. steak at 48¢.

5. $4\frac{1}{4}$ lb. beef at 35¢.
$1\frac{7}{8}$ lb. pork chops at 30¢.
$3\frac{1}{2}$ lb. bacon at 32¢.
$2\frac{3}{4}$ lb. steak at 48¢.

6. 2 pk. potatoes at 49¢.
5 lb. onions at $4\frac{1}{2}$¢.
$\frac{1}{2}$ doz. oranges at 25¢.
$\frac{1}{4}$ lb. cream of tartar at 65¢.
$2\frac{3}{8}$ lb. butter at 42¢.
8 lb. sugar at $9\frac{1}{2}$¢.

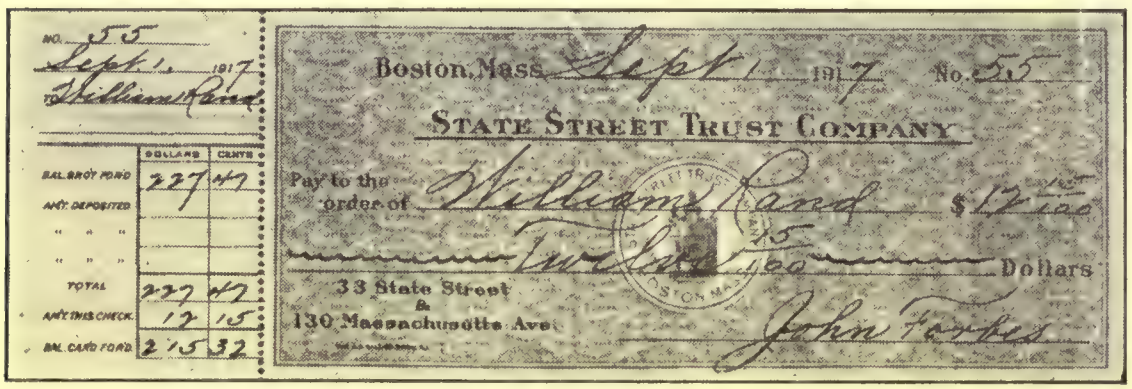
7. 18 spools of thread at 5¢.
8 yd. gingham at $12\frac{1}{2}$¢.
5 yd. silk at 89¢.
$3\frac{1}{2}$ yd. serge at 79¢.

8. $2\frac{1}{2}$ doz. spools of thread at 30¢.
3 yd. velvet at \$3.50.
$22\frac{1}{2}$ yd. braid at 3¢.
9 yd. silk at $87\frac{1}{2}$¢.

Find the change if \$20 is given in payment of this bill.

§ 26. Checks. Bills are generally paid either by cash or by check.

A ***check*** is a written order on a bank to pay money. The form of a check is the following:



John Forbes must have money in the State Street Trust Company in order to write a check on this bank. Money put in banks is termed *deposits*, and all individuals who have money in the bank are called *depositors*.

When William Rand receives the above check from John Forbes he may do either of two things:

(*a*) Deposit it, or cash it, at the bank in which he has a deposit.

(*b*) Make the check payable to a second party by writing on the reverse side "Pay to the order of ——."

In either case he must *indorse* the check. That is, he, William Rand, must sign his name across the back exactly as it is written on the face of the check. This is an acknowledgment of his having received the value of the check. The indorsed check is a valid receipt to John Forbes.

Do not indorse a check until you are ready to cash it. If you wish to mail a check for deposit, write above your indorsement, "For deposit only."

In making out checks the following points should be noted:

(*a*) First, fill out the stub.

(*b*) Write the check, making all items the same as those on the stub.

(*c*) On the check, write the amount both in figures and in words.

EXERCISES

1. On Feb. 1, 1918, Charles M. Jones paid John K. Francis $28.50 by check. He had a balance of $142.75 on deposit at the Market Street Trust Co., Philadelphia, Pa. Write the check.

2. On Dec. 12, 1917, Frank K. Little paid William R. Dow $13.12 by check. He had a balance of $93.34 on deposit at the Cosmopolitan Trust Co., New York City. Write the check.

3. On Jan. 3, 1918, R. M. Eaton & Co. paid the Johnson Furniture Co. $374 by check. They have a balance of $4762.45 on deposit in the Security Trust Co., Chicago, Ill. Write the check.

Using the name of a bank with which you are familiar, write checks for the payment of the following bills:

Assume that you have a balance of $75 on deposit.

4. Bill, Example 1, page 58.

5. Bill, Example 2, page 58.

6. Bill, Exercise 1, page 59.

7. Bill, Exercise 2, page 59.

8. Bill, Exercise 3, page 59.

9. Bill, Exercise 4, page 59.

10. Bill, Exercise 5, page 59.

11. Bill, Exercise 6, page 59.

IV. Investments — Stocks and Bonds

§ 27. Investments. By a study of accounts and budgets you have noticed the increased earning power of an individual, or family, that results from spending according to a systematic plan.

It is also essential for you to appreciate the importance of investing wisely the money you have saved.

For the ordinary person, who is able to save from time to time only small amounts, it is true that the Savings Bank is the safest place to deposit or invest such savings; and you have observed already, in this course, how small sums of money which have been placed in Savings Banks or other banks that pay interest on deposits increase in value as the years go by.

There are, however, two classes of investments in the business world which are generally known and which are quite safe. Either of these may assure to the investor a higher rate of interest than that paid by the Savings Bank or by other interest paying banks. These are known as: (1) Stocks and (2) Bonds. Each of these will be treated briefly.

§ 28. Stocks. When several persons desire to go into business together and act as one individual, it is necessary for them to be so organized under the law of some state as a *Company* or *Corporation.*

The total amount of money invested in such a *Company* or *Corporation* is called its *Stock* or *Capital.*

The stock is divided into *shares*, and each share, when the company is organized, has a definite value; such as $100, $50, $25, or $10. The original value of a share is called its *par value.*

A *stockholder* of a company is one who owns one, or more, of its shares of stock. All such persons together are called the stockholders; they usually control the company.

A *Stock Certificate* is a formal statement issued by the company, showing the number of shares owned by the person whose name is written on the face of the certificate.

Quite frequently a company is organized so that it has two kinds of stock: *preferred stock*, which pays a certain fixed rate of interest, when the profits permit; *common stock*, which pays whatever rate of interest is possible after all other demands have been met. The voting power is often restricted to the holders of common stock.

In examining a certificate of stock the following points should be noticed:

(*a*) Legal name of company.

(*b*) State in which the company is organized.

(*c*) Kind of stock — preferred or common.

(*d*) Date of ownership.

(*e*) Name of owner.

(*f*) Number of shares owned.

(*g*) Par value of a share.

All money which a company pays to the stockholders from its income is called *dividends*.

Stocks must be bought or sold through a broker. The broker's commission (which is called the *brokerage*), for buying, or for selling, is usually $\frac{1}{8}\%$ of the *par* value of the stock. The price for which stock sells is called its *market value*.

The market value of important stocks is published daily in the newspapers and varies according to conditions in the business world.

EXERCISES

1. The U. S. Rubber Co. pays dividends quarterly. The rate of interest on the first preferred stock is 8%. A owns 5 shares of this stock. How much will he receive in dividends at the end of each quarter? At the end of each year?

SPECIMEN SOLUTION.

\$500	(par value of the 5 shares)
.02	(a rate of 8% a year is 2% a quarter)
\$10.00	(dividend at the end of a quarter)
4	(4 quarters a year)
\$40.	(dividend for a year)

Ans. A receives \$10 each quarter. A receives \$40 a year.

NOTE. The arithmetic involved in finding the amount of dividends on an investment in stocks is the same as the arithmetic of simple interest. The rate of interest paid is always on the *par value* of the stock, not the market value. The par value of the stocks represented here is \$100 unless otherwise stated.

2. I own 5 shares of American Telephone and Telegraph, which pays 8%. How much do I receive in dividends for each quarter? How much for a year?

3. B owns 7 shares of American Woolen, first preferred, which pays 7%. How much does he receive in dividends for each quarter? How much for a year?

4. A owns 10 shares of General Electric, which pays 8%. What income does he receive from this for each quarter? What income for a year?

5. I own 8 shares of U. S. Steel, first preferred, which pays 7%. What income do I receive from this investment for a quarter? For a year?

6. B owns the following: 7 shares of American Tel. & Tel., 8%, and 3 shares of U. S. Steel, first preferred, 7%. What is his total income from these two investments for each quarter?

7. A owns the following: 15 shares of General Electric, 8%, and 7 shares of American Woolen, 1st pfd., 7%. What is his total income from these two investments for a year?

8. B owns 10 shares of Reece Buttonhole. This stock pays 12%. The par value of a share is \$10. What does B receive in dividends for a year from this investment?

If the following stocks are purchased at the market value quoted, brokerage $\frac{1}{8}$%, what is the actual rate of income received from each?

9. American Tel. & Tel., market value 106, rate 8%. (Income = \$8. Rate of income = $8 \div 106\frac{1}{8}$.)

10. General Electric, market value $124\frac{7}{8}$, rate 8%.

11. American Woolen, 1st pfd., market value 88, rate 7%.

12. U. S. Rubber, 1st pfd., market value 108, rate 8%.

§ 29. Bonds. A ***bond*** is a written promise to pay a specified sum of money at a stated time. It must be signed by the maker and it must bear his seal.

The most common kind of bond is that issued by a corporation, or by a government, when it wishes to borrow money.

Such bonds run for a term of years and bear a fixed rate of interest which is payable at stated intervals.

Government bonds are of two classes, *registered* and *coupon*.

In the case of the *registered bond* the interest is sent to the holder of the bond as it becomes due; in the case of the *coupon bond*, the coupons (certificates which state the amount of interest due) must be cut from the bond and sent to the government for collection. Examine a U. S. Government Liberty Bond.

In examining a bond, the following points should be noticed:

(*a*) Name of corporation or government.
(*b*) Par value of the bond.
(*c*) Date of purchase.
(*d*) Kind — registered, or coupon.
(*e*) Name of owner.
(*f*) Rate of interest.
(*g*) When interest is payable.
(*h*) Number of years before the bond matures.

Since bonds are secured by a legal claim on the property of the corporation issuing them, and since they guarantee a fixed rate of interest, they are very safe investments.

The legal claim on the property of a person, or corporation, is known as a ***mortgage***. In case of a failure to pay the claim, the holder of the mortgage may sell the property. The arithmetic of bonds and mortgages corresponds to that of simple interest.

EXERCISES

1. Mr. Rickard bought a \$50 U. S. Liberty Bond for each of his three children. The rate of interest is $3\frac{1}{2}\%$ and the interest is payable twice a year. What was the cost of the three bonds? How much interest will be due on these bonds at the end of the first six months?

2. Mr. Vern and his wife each bought a $100 U. S. Liberty Bond of each of the first two issues. The rate of interest on the first was $3\frac{1}{2}\%$; on the second, 4%. What was the total amount paid for these bonds? What amount of interest will be due at the end of six months on the bonds of the first issue? Of the second issue?

3. The Alpha society purchased eight $100 U. S. Liberty Bonds, first issue, $3\frac{1}{2}\%$. How much interest will be received at the end of every six months?

4. The Dixton Company purchased $25,000 of the U. S. Bonds paying $3\frac{1}{2}\%$. How much interest will be received annually on this investment?

5. A takes $500 from the Savings Bank, which pays 4% interest, and invests it in Liberty Bonds paying $3\frac{1}{2}\%$. How much does he lose during the first six months?

6. A exchanges $500 worth of Bonds paying $3\frac{1}{2}\%$ for $500 worth of Bonds paying 4%. How much better return does he get on the new investment each year?

7. A certain school decided to purchase $500 of Liberty Bonds paying 4% and donate all interest received from this investment to the Red Cross. How much will be given to the Red Cross every six months?

8. A certain company invested $25,000 in Liberty Bonds of the first issue, $3\frac{1}{2}\%$, and $100,000 in Liberty Bonds of the second issue, 4%. What is the total amount of interest received annually from these two investments?

9. I desire to contribute $28 a year to the Red Cross. How many $100 Liberty Bonds, paying 4%, will I need to buy in order that the income from these may provide the required amount?

10. A man who is about to enlist desires to provide an income from investments of $600 a year for his family. If he buys 4% U. S. Government Bonds, what will be the face value of the bonds that he will need to buy?

11. There is a mortgage for $4000 at 5% on Mr. Brown's farm. How much interest does Mr. Brown pay annually on this mortgage? Mr. Brown sells hay from this farm in October, 1917, for $16 a ton. How many tons of hay will have to be sold in 1917 to pay the interest on the mortgage?

12. There is a mortgage for $600 at 6%, on the live stock on A's farm. How much interest does A have to pay annually? To pay the interest, A sells a cow for $45. How much will he have left after paying the interest?

13. A buys a pair of horses for $600. He pays $125 in cash and gives his note, interest at 6%, secured by a mortgage on his farming utensils, for the balance. What is the face of the note? How much will A have to pay to settle the note at the end of a year?

14. A buys an automobile for $650. He pays 20% of this in cash and gives his note for the balance, with interest at 6%, secured by a mortgage on the automobile with the following agreement: To pay at the end of three months, interest due and 25% of the face of the note; to pay at the end of six months, interest due and 50% of the balance; to pay at the end of one year, interest due and balance on note.

Find the amount that must be paid (1) at the end of three months, (2) at the end of six months, (3) at the end of the year.

CHAPTER IV

ARITHMETIC OF THE FARM AND THE CITY

I. Arithmetic of the Farm

§ 30. Farm Accounts. In order that the farmer may determine whether he is conducting his farm on a paying basis or not, he should keep a record of his total receipts and of his total expenditures during the year.

Below is shown a form of keeping such a record.

Farm Account

Receipts				Expenditures			
1917				1917			
Jan. 1	Cash on hand	$ 45	50	Jan. 3	1 ton bran	$ 30	00
5	1 calf	15	40	7	4 pigs at $3.50	14	00
7	12 doz. eggs at 40¢	4	80	12	Repairs on sled	7	50
8	10 tons hay at $12	120	00	15	Wages, hired man	12	00
15	Milk	50	25	21	Taxes on farm	76	25
24	5 tons straw at $7	35	00	31	Wages, hired man	12	00
				31	Balance	119	20
		270	95			270	95
Feb. 1	Balance	119	20				

EXERCISES

1. The record on a farm for the month of February was as follows:

Receipts. — Balance, $119.20; 2 calves at $13.60; 30 doz. eggs at 38¢; milk, $75.60; 45 bu. oats at 42¢; 1 colt, $125.

Expenditures. — Groceries, $14.25; wages, $24; 2 tons mixed feed at $35; 2 tons coal at $6.10; grass seed, $45.

Rule a sheet as shown in the model, and enter the above transactions in it. Find the balance for February.

2. The record on the same farm for March was as follows:

Receipts. — Balance ? (Ex. 1); 1 cow, $42; 42 doz. eggs at 35¢; milk, $92.60; 2 tons hay at $12; 2 hogs at $15.50; 2 calves at $12.50; 8 bu. beans at $4.20.

Expenditures. — 1 ton mixed feed, $38; 1 ton bran, $31; wages, $30; sawing wood, $12; 25 bu. seed grain at 75¢; 2 tons coal at $6.10; horseshoeing, $15.80; groceries, $20.60; insurance, $10.80.

Enter the items as in Ex. 1, and find the balance for March.

3. The record on the same farm for the month of April was as follows:

Receipts. — Balance ? (Ex. 2); 60 doz. eggs at 28¢; milk, $128.40; 5 calves at $11.20; 5 tons hay at $14.

Expenditures. — 2 tons fertilizer at $36; 2 tons mixed feed at $38; wages, $32; 4 bu. seed corn at 95¢; harrow, $26.50; horse, $125; groceries, $18.20.

Enter the items as in Ex. 1, and find the balance for April.

4. The record for the same farm for March was:

Receipts. — Balance ? (Ex. 3); 35 doz. eggs at 28¢, 40 doz. eggs at 25¢; milk, $121.40; 2 calves at $10.75.

Expenditures. — 1 ton fertilizer at $35.50; $2\frac{1}{2}$ tons feed at $37.50; wages, $46.50; 1 cow at $65.00; groceries, $23.75. Enter the items as in Ex. 1, and find the balance for March.

§ 31. Accounts for Special Crops.

When a farmer wishes to know whether he is making or losing on a particular crop, it is necessary for him to keep a special account for this crop.

The following is an account of the total cost of production of, and the receipts from, 12 acres of rye:

		Expenditures	Receipts
1916			
Aug. 24	Plowing 12 acres at $2.25	$27.00	
Sept. 5	Harrowing and sowing	9.00	
Sept. 5	Seed grain, 24 bu. at $1	24.00	
Sept. 5	Fertilizer, 3 tons at $32	96.00	
1917			
July 5	Reaping and shocking	18.00	
July 11	Threshing 270 bu. at 4¢	10.80	
July 11	Labor	12.00	
July 11	Pressing straw, 6 tons at $3	18.00	
Aug. 12	Hauling grain to market	4.50	
Aug. 12	Sold 260 bu. at $1.25		$325.00
Aug. 15	Hauling straw to market	6.75	
Aug. 15	Sold straw, 6 tons at $15		90.00
Dec. 1	Interest and taxes	21.60	
	Totals	$247.65	$415.00
	Profit, or balance	167.35	
		$415.00	$415.00

EXERCISES

1. The following is an account of the total cost of production of, and the receipts from, 25 acres of wheat:

Expenditures. — Aug. 16, 1916, plowing 25 acres at $2.25; Aug. 21, harrowing and sowing, $21; Aug. 21, seed grain, 50 bu. at $1; Aug. 21, 8 tons fertilizer at $32; July 8, 1917, reaping and shocking, $38; July 20, threshing and labor, $30; July 20, pressing straw, $45; Sept. 5, delivery of grain to market, $25; delivery of straw, $18.

Receipts.—Sept. 12, 525 bu. at $1.15; Sept. 12, 15 tons of straw at $6.

Rule a sheet as shown in the preceding model, and enter the above transactions in it. Find the profit on this crop of wheat.

2. The following is an account of the total cost of production of, and the receipts from, sixteen acres of beans:

Expenditures. — Plowing 16 acres at \$2.25; harrowing and planting, \$18; seed, \$35; fertilizer, \$128; cultivating and hoeing, \$56; pulling, \$24; threshing, \$26; delivery to market, \$10.

Receipts. — 256 bu. at \$2.85; 8 loads of bean vines at \$5 a load.

Proceed as in Ex. 1, and find the profit on this crop of beans.

3. The following is an account of the total cost of production of, and receipts from, 12 acres of oats:

Expenditures. — Plowing 12 acres at \$2.25; harrowing and sowing, \$16; 25 bu. seed oats at 55¢; 3 tons fertilizer at \$28; reaping and threshing, \$36.60.

Receipts. — 336 bu. at 48¢; 9 tons straw at \$5.50.

Proceed as in Ex. 1, and find the profit on this crop of oats.

4. The following is an account of the total cost of production of, and the receipts from, forty-eight acres of potatoes:

Expenditures. — Plowing 48 acres at \$2.25; harrowing and planting, \$85; seed, \$388; 20 tons fertilizer at \$28; cultivating and hoeing, \$75; spraying, \$60; digging and storing, \$380; delivery to market, \$150.

Receipts. — 1200 bu. at 95¢; 1600 bu. at \$1.15; 500 bu. at \$1.45; 800 bu. at 52¢.

Proceed as in Ex. 1, and find the profit on this crop of potatoes.

FARM PROBLEMS

1. A farmer sold during the year the following products: 25 tons of hay at \$12.50 a ton; 9 tons of hay at \$11.50 a ton; 16 tons straw at \$6.50 a ton; 75 bu. buckwheat at 58¢ a bushel; 125 bu. oats at 43¢ a bushel. Find the total receipts from these sales.

2. The receipts from a poultry farm for April were as follows: 883 doz. eggs at 34¢ a dozen; 227 doz. eggs at 45¢ a dozen; 75 hens at 95¢ each; 235 chicks at 9¢ each. Find the total.

3. The receipts from a dairy for the month of May were as follows: 4495 qt. of milk at 9¢ a quart; 225 pt. of cream at 27¢ a pint; 3 calves at \$2.75 each; 2 calves at \$12.50 each; 2 cows at \$76 each. Find the total.

4. The record of milk from one cow in a dairy for 15 consecutive days was as follows: 54 lb.; 52 lb. 12 oz.; 55 lb. 4 oz.; 56 lb. 8 oz.; 54 lb. 2 oz.; 53 lb. 12 oz.; 54 lb. 2 oz.; 54 lb. 4 oz.; 56 lb. 4 oz.; 55 lb. 14 oz.; 56 lb. 6 oz.; 54 lb. 8 oz.; 55 lb. 12 oz.; 53 lb. 2 oz.; 54 lb. Find the total number of pounds and the average number per day.

5. The weights of 8 bags of oats were as follows: 76 lb. 8 oz.; 78 lb. 4 oz.; 79 lb.; 72 lb. 14 oz.; 81 lb.; 76 lb. 12 oz.; 70 lb. 12 oz.; 78 lb. 8 oz. Find the total weight and the average weight.

6. The weights of 7 hogs were as follows: 324 lb.; 239 lb.; 276 lb.; 293 lb.; 383 lb.; 297 lb.; 357 lb. Find the total weight and the average weight.

7. A farmer mixed a ton of fertilizer as follows: 40% of cottonseed meal at \$36 a ton; 57% of acid phosphate at \$18 a ton; 3% of muriate of potash at \$50 a ton. What did the ton of fertilizer cost?

8. The amount of milk delivered at a creamery by a farmer for the month of May was as follows:

1. 343 lb.	9. 359 lb.	17. 397 lb.	25. 447 lb.
2. 337 lb.	10. 355 lb.	18. 417 lb.	26. 442 lb.
3. 345 lb.	11. 350 lb.	19. 425 lb.	27. 438 lb.
4. 347 lb.	12. 352 lb.	20. 423 lb.	28. 452 lb.
5. 341 lb.	13. 347 lb.	21. 419 lb.	29. 476 lb.
6. 335 lb.	14. 349 lb.	22. 417 lb.	30. 478 lb.
7. 354 lb.	15. 357 lb.	23. 425 lb.	31. 472 lb.
8. 351 lb.	16. 385 lb.	24. 443 lb.	

Find the total number of pounds delivered. The price received at the creamery was \$1.95 per 100 lb.; find the amount received for milk for the month of May.

9. If 5.6 pounds of butter were made from every 100 pounds of milk, how many pounds of butter were made from the milk delivered in Ex. 8? If this butter was worth 36 cents a pound, what was the value of the butter?

10. A chemical analysis of a pound of butter showed 82.6% of fat; .9% of casein; 2.9% of salt; 13.6 % of water. Find correct to a tenth of a pound the weight of each substance in a tub containing 56 lb. of butter.

11. The composition of a pound of alfalfa meal is as follows: 9.1% of water; 9.5% of ash; 26.6% of fiber; 36.8% of nitrogen; 2.1% of fat; 15.9% of protein. Find the weight of each substance in 1 ton (2000 lb.) of alfalfa meal.

12. If the average yield of wheat per acre in the United States is 12.1 bu., and in Great Britain 31 bu., what per cent greater is the yield in Great Britain than in the United States? What per cent less is the yield in the United States than in Great Britain?

13. When clover seed is \$12 a bushel and 17% of it is weed seed and 8% of the clover seed will not germinate, what is the actual cost of a bushel of good seed?

14. A bushel of seed corn was bought for 95 cents; a germination test showed that 5% of it would not germinate. At this rate what was the actual cost of a bushel of good seed?

15. When the price of oats is \$1.50 per 100 lb., corn \$2.20 per 100 lb., bran \$32 a ton, and hay \$12 a ton, what will be the cost of a day's feed for a horse, if he is fed 8 lb. of oats, 4 lb. of corn, 8 lb. of bran, and 14 lb. of hay?

16. Using the result in Ex. 15, determine the cost of feeding a team of horses for 1 year.

17. An ordinary cow is supposed to give about 15 lb. of milk a day for 300 days in the year. If the milk tests 4.2% butter fat and the butter averages 31 cents a pound, what is received for the butter from a cow in a year?

18. If a cow eats, in a year, 2.5 tons of hay worth \$9, 1200 lb. feed worth \$32 a ton, and pasture amounting to \$8, what is the cost of keeping the cow for a year?

19. A mower, costing \$45 and cutting on the average 28 acres a year, will last about 14.8 years. Interest on money invested and the repairs amount to \$42. If this mower is actually in use for 50 days of its life, what is the cost of the mower per day? What is the cost per acre of grass cut?

20. A binder, costing \$135 and cutting 34.5 acres annually, lasts about 16.2 years. Interest on the money invested and the repairs amount to \$102.50. If the binder is actually in use for 45 days of its life, what is the cost of the binder per day? Per acre cut?

II. Public Utilities

§ 32. The Cost of Gas.

A. Reading the gas meter.

The dials of a gas meter are shown in Fig. **3.**

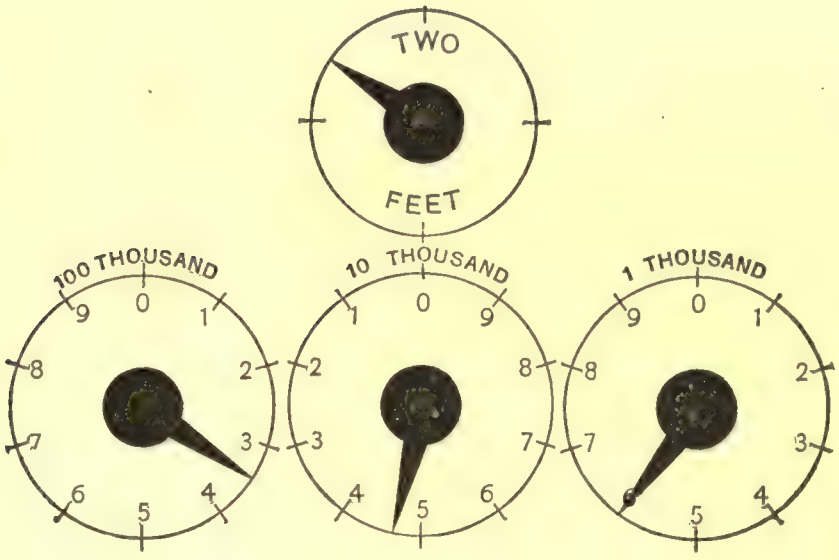


Fig. 3. — Dials of Gas Meter.
Readings are in thousands of cubic feet.

Each division on the right-hand circle denotes 100 feet; on the center circle 1,000 feet; and on the left-hand circle 10,000 feet.

Read from left-hand dial to right, always taking the figures which the hands have passed. If the hand is very near a figure, whether that figure or the next lower is to be taken can be determined by observing the hand in the next lower circle. Hence, by the above dials, read 3, 4, 6 and add two ciphers for the hundreds, making 34,600 feet registered.

EXERCISES

See if you can read the number of feet registered in each of the Figs. 4 and 5.

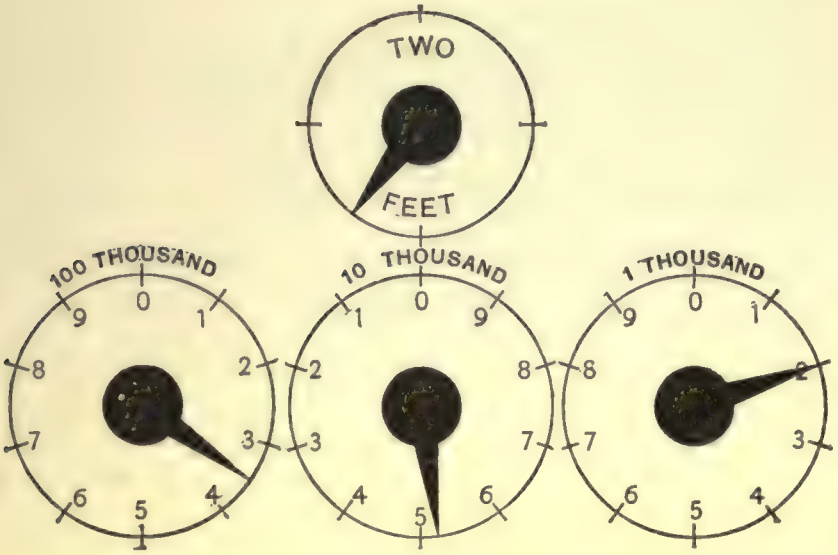


Fig. 4.

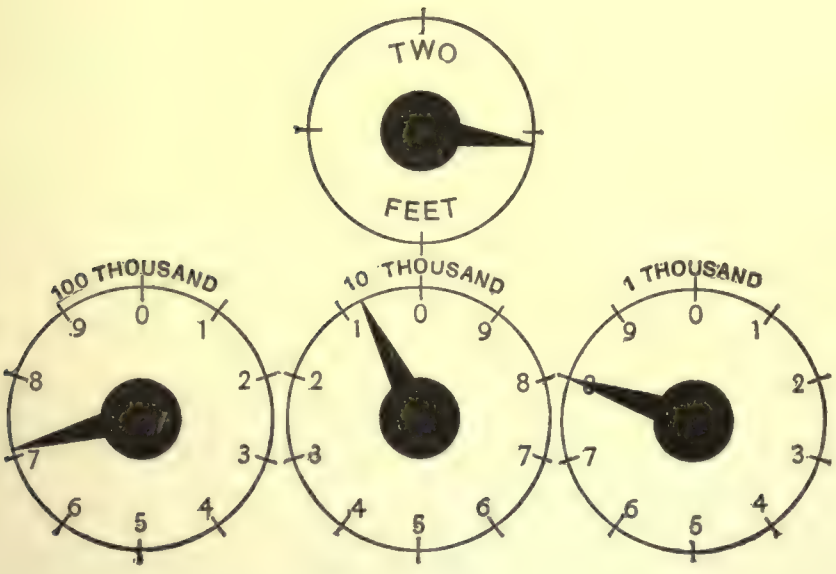


Fig. 5.

B. Finding the amount of the gas bill.

A form of bill is shown below:

1917 — 6 — 1

JAMES FRENCH,
12 Oak Place,
NEWTON, MASS.

To Newton & Watertown Gas Light Co., Dr.
For gas supplied at 90 cents per 1000 cubic feet

Balance unpaid as per account rendered		
State of meter on this date	13400	
State of meter on date of last bill	11100	
Cubic feet	2300	
		$2.07

EXERCISES

Find the amount of each of the following gas bills, *for gas supplied at* 90 *cents per* 1000 *cubic feet.* In each case find first the total number of cubic feet used, as in the preceding illustration.

1. State of meter on this date 09100
   State of meter on date of last bill 06700
   Cubic feet

2. State of meter on this date 04000
   State of meter on date of last bill 02100
   Cubic feet

3. State of meter on this date 10100
   State of meter on date of last bill 08300
   Cubic feet

4. State of meter on this date 023800
   State of meter on date of last bill 021100
   Cubic feet

5. State of meter on this date	026100
State of meter on date of last bill	023800
Cubic feet	
6. State of meter on this date	032100
State of meter on date of last bill	029400
Cubic feet	

C. Cost of gas consumed per hour.

Figure 6 on page 80 shows the cost, in mills per hour, of various styles of gas-burners, on the basis of gas *at* \$1 *per* 1000 *cubic feet.* The large difference between maximum and minimum costs is due to difference in size of burners and difference in the pressure of the gas supply. The mean value represents the cost under average conditions.

For a water heater of ordinary household size the gas would cost from 3 to 8 cents per hour. For larger automatic water heaters gas would cost from 10 to 25 cents per hour for the period that the heater is in operation. The gas for the pilot flame for these heaters costs one tenth of a cent per hour. Room heaters require 5 to 12 cents worth of gas per hour. Since a match costs only one tenth of a mill or less, it is usually economical to turn off the gas when not in use, except for short intervals.

EXERCISES

NOTE. In each of the following exercises use the *mean cost* given in Fig. 6, p. 80.

1. During the month of January the gas light in a room is used for an average of $4\frac{1}{2}$ hours per day; an inverted mantle is used. What is the cost of gas for January?

2. The pilot flame on a water heater costs one tenth of a cent per hour. How much does it cost to burn it during the months of June, July, and August?

3. An inverted mantle costs 75¢. Three inverted mantles are bought to replace the fishtail burners. When all three burners are in use, what amount is saved per hour? By the saving thus effected how many hours use will pay for the three inverted mantles?

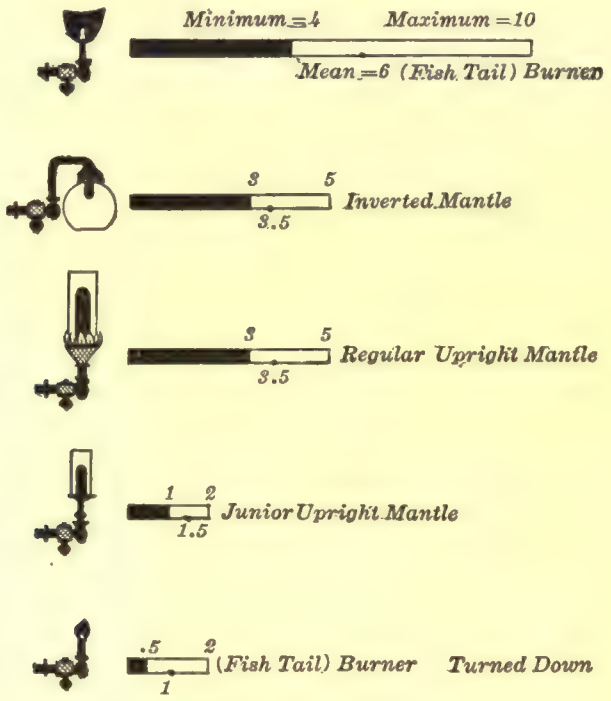


FIG. 6. COST OF GAS USED IN VARIOUS BURNERS.

The cost is given in mills per hour, on the basis of gas at $1.00 per thousand cubic feet. The maximum and minimum are stated above, and the mean below, the horizontal line graph, in each case.

4. If a junior upright mantle is used in the bathroom in place of the fishtail burner, what is the saving per hour? What is the saving from Dec. 1 to March 15, inclusive, if the gas is burning on an average of $1\frac{1}{2}$ hours each day?

5. If a junior upright mantle is used in the hall in place of the regular upright mantle, what is the saving per hour? What is the saving from Nov. 1 to March 1, if the gas is burning on an average of $2\frac{1}{4}$ hours each day?

6. The cost of heating water by gas is 5.5¢ per hour. What saving is effected by having the water boiler connected with the furnace, if the furnace is heated from Oct. 1 to May 15 and the gas heater would be used on an average of 40 min. each day during this period? What would be the additional cost of gas if the pilot flame were kept burning all this time?

7. The cost of heating water by the automatic gas heater is 17.5¢ per hour. What saving is effected by having the water boiler connected with the furnace, if the furnace is heated from Oct. 1 to May 15, and the gas heater would be used on an average of 50 min. each day during this period?

8. The cost of heating a certain room by a gas heater is 8.5¢ per hour. What is the cost for January, if the heater is running on an average of 6 hours per day?

9. The coal for heating a certain eight-room house by furnace for 8 months of the year cost $80. (*a*) What was the average cost per room for heat for a month? (*b*) Estimating the cost of heating the room in Ex. 8 by furnace at $\frac{7}{5}$ the average cost in (*a*), how much cheaper was it to heat the room by furnace than by gas?

10. The following gas light equipment is used on the first floor of a house during the month of January: kitchen, 1 regular upright mantle, used 2 hr. per day; dining room, 1 inverted mantle, used $\frac{3}{4}$ hr. per day; living room, 2 inverted mantles used $2\frac{3}{4}$ hr. per day; hall, 1 junior upright mantle, used 4 hr. per day. If gas is 90¢ per 1000 cu. ft., compute the gas bill for January.

§ 33. The Cost of Electricity.

A. Reading the electric meter.

In Fig. 7 are shown the dials of a modern electric meter. The method of reading is similar to that of reading the

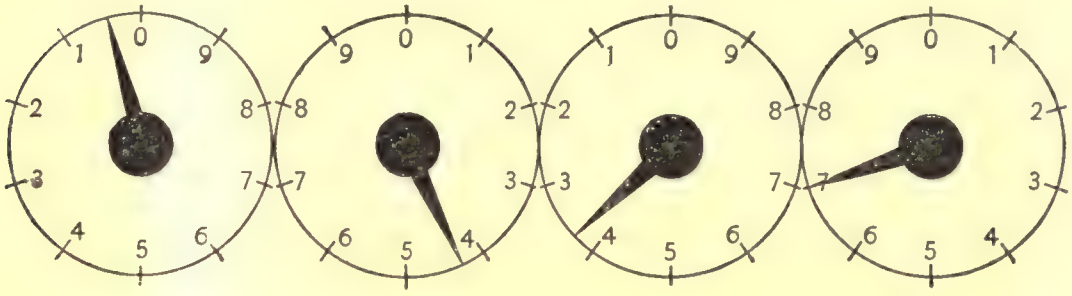


FIG. 7. — ELECTRIC METER — KILOWATT HOURS.

dials of a gas meter. In taking down the figures, notice the direction in which the pointer is turning and write down the figure that the pointer has just passed. Hence, in Fig. 7, write 4, 3, 7. The reading is 437 kilowatt hours.

See if you can read the dials shown in Figs. 8 and 9.

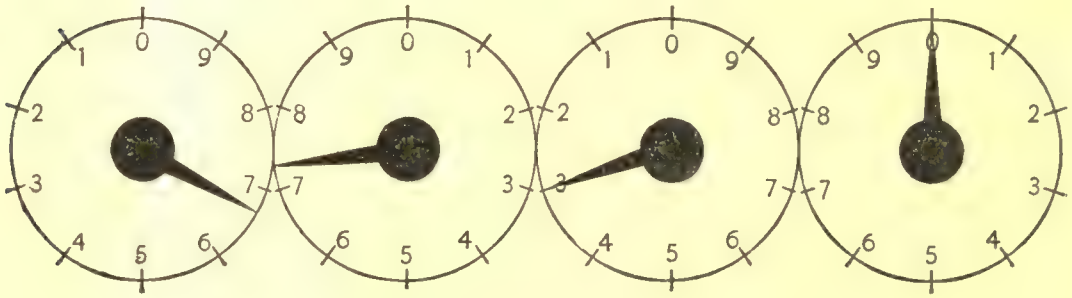


FIG. 8. — KILOWATT HOURS

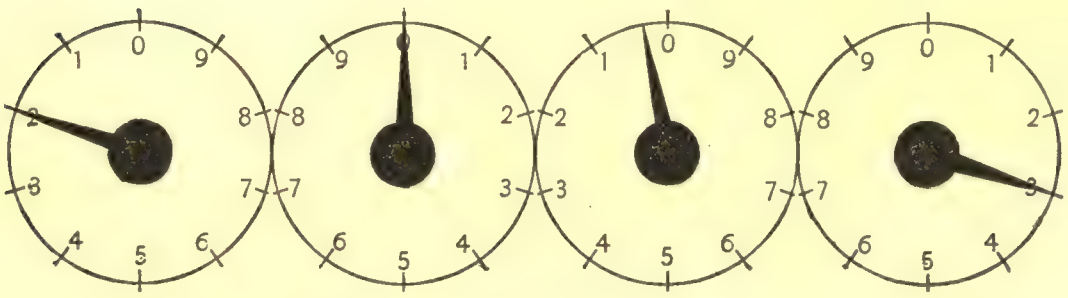


FIG. 9. — KILOWATT HOURS

B. Electric units.

A ***watt*** is the unit of measure of electric power. A 15-*watt* electric light bulb is one that requires a power of 15 watts to keep it burning; a 25-watt light requires a power of 25 watts; etc. A ***kilowatt*** is 1000 watts.

C. Sample of a bill for electricity used.

THE EDISON ELECTRIC ILLUMINATING COMPANY OF BOSTON

JAMES FRENCH,
10 Oak Place, NEWTON, MASS.

For electric service from — March 20 to April 18, 1917

219 Present Meter Reading		
205 Previous Meter Reading		
14 Kilowatt Hours at 10 cents	1	40
Received payment for the Company 1917		

D. The cost of electricity at 10 cents per kilowatt hour.

APPARATUS	WATTS USED	COST IN CENTS PER HOUR
1000-watt light (kilowatt light) . . .	1000	10
100-watt light	100	1
60-watt light	60	$\frac{3}{5}$
40-watt light	40	$\frac{2}{5}$
25-watt light	25	$\frac{1}{4}$
Coffee percolator	450	$4\frac{1}{2}$
6-pound iron	550	$5\frac{1}{2}$
Radiant toaster	600	6
8-inch fan (full speed)	25	$\frac{1}{4}$
12-inch fan (full speed)	60	$\frac{3}{5}$
Sewing machine motor	55	$\frac{1}{2}$
Vacuum cleaner	$133\frac{1}{3}$	$1\frac{1}{3}$
1 horse power motor (full load) . . .	746	$7\frac{1}{2}$

EXERCISES

1. In my desk lamp there is a 40-watt bulb. If I use this lamp on an average of $2\frac{1}{2}$ hours for four evenings a week, what is the cost of the electricity used? What is the cost for 200 evenings?

2. In a class room there are six 60-watt bulbs. What is the cost per hour for electricity if all the bulbs are turned on? In December it was necessary to turn on the lights in this room for 12 days from 2.45 to 4.00 P.M. What was the total cost of the electricity used?

3. In a living room there are 4 lights on the chandelier; two of these are 40-watt bulbs and two are 25-watt bulbs. What is the cost per hour when all are turned on? How much would be saved each evening, if only the two 40-watt bulbs were turned on and they were used for 4 hours each evening?

4. How much does it cost father each night for electricity when he forgets to turn off the 25-watt light in the cellar, if it is left from 10 P.M. till 7 A.M.?

5. It costs $2.25 to install a switch with a warning light for the cellar. In how many hours would the electricity used in a 25-watt bulb cost this amount, if the light were left turned on?

6. A 6-pound iron (550 watts) is used on an average of 4 hours each week. What is the cost of the electricity used for one year?

7. An electric coffee percolator (450 watts) is used each morning for 30 minutes. What is the cost for one year?

8. An electric radiant toaster (600 watts) is used for 20 minutes each morning. What is the cost for one year?

Compare this with the cost of toasting bread by gas, if the time is the same, and if the gas burner costs 1.5¢ per hour.

9. A sewing machine motor (55 watts) is used for two hours each day for 6 days. What is the cost of the electricity used?

10. A vacuum cleaner (133⅓ watts) is used on an average of three hours per week. What is the cost of the electricity used for one year?

11. An 8-inch fan runs at full speed (25 watts) from 9 o'clock until 5 o'clock in an office for 20 days during the month of July. What is the cost of the electricity used? What would be the cost for the same period of time if a 12-inch fan (60 watts) were used?

12. In a class room, two 12-inch fans run at full speed from 10 o'clock until 4 o'clock for three consecutive days. What is the cost of the electricity used?

13. In a shop a 1-horse power motor (746 watts), carrying a full load, is used for 6 hours daily for 10 days. What is the cost of the electricity used?

14. On a certain day, the following are used, each for the length of time given: radiant toaster, 30 min.; coffee percolator, 45 min.; 6-pound iron, 2 hr.; vacuum cleaner, 1½ hr.; sewing machine motor, 1½ hr. Find the total cost of the electricity used.

15. An old-fashioned carbon filament light that uses 60 watts may be replaced without loss of light by a new tungsten filament that uses 25 watts. If the new light costs 35¢, how many hours must it be burned in order to pay for itself, if electricity costs 10¢ per kilowatt watt?

§ 34. The Cost of Water.

A. Reading the water meter.

The dial of a water meter is shown in Fig. 10. The number on the outside of each circle indicates the number of cubic feet for one complete revolution of the hand. The small dial at the left, measuring 1 cu. ft. for a complete revolution, is disregarded in reading the meter; it is used for testing purposes.

In reading the dials, take the figure that the hand has just passed, writing the figures from left to right. If the hand is very near a figure, whether that figure, or the next lower, is to be taken can be determined by observing the hand in the next lower circle. Hence, in Fig. 10, we read 11,867 cubic feet.

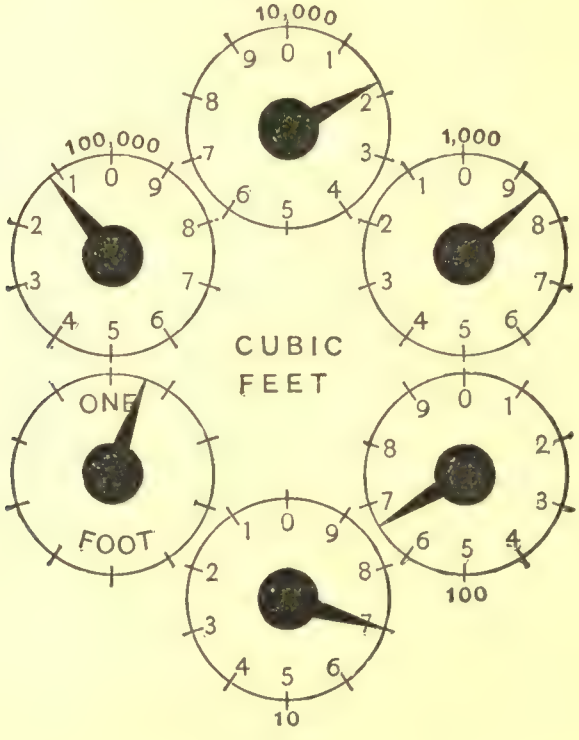


Fig. 10.

One cubic foot is equal to $7\frac{1}{2}$ gal. Hence, to reduce a meter reading in cubic feet to gallons, multiply the number of cubic feet by $7\frac{1}{2}$.

EXERCISES

See if you can read the number of cubic feet registered in each of the Figs. 11 and 12.

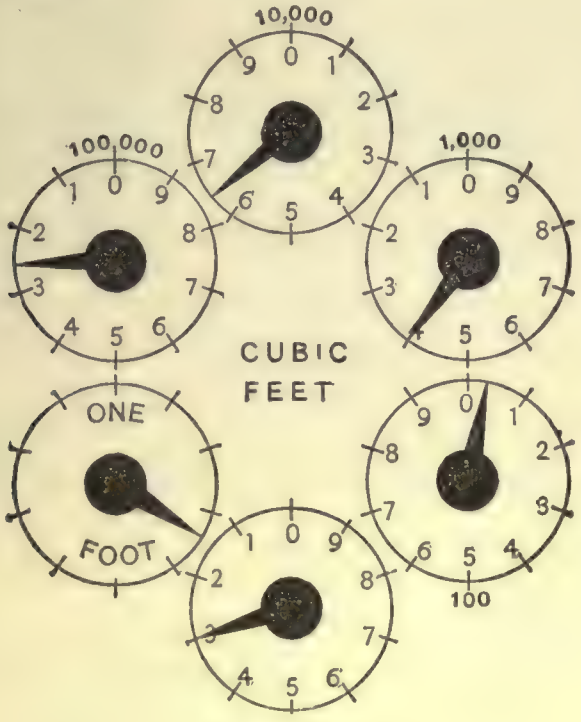


Fig. 11.

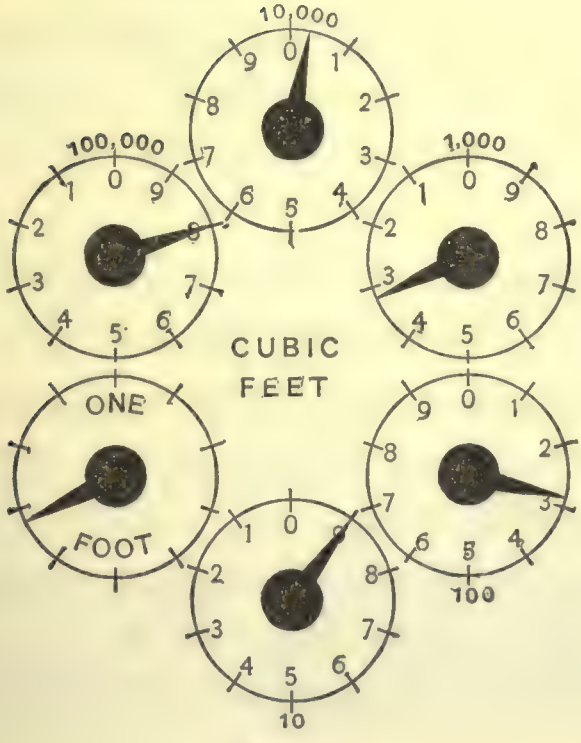


Fig. 12.

B. Cost of water at \$3.60 *per* 100 *cubic feet*

Stream	Gallons per Day	Cost per Day
1 inch	3225	\$15.48
$\frac{3}{4}$ inch	2812	13.50
$\frac{1}{2}$ inch	1596	7.66
$\frac{1}{4}$ inch	473	2.27
$\frac{1}{8}$ inch	170	.82
$\frac{1}{16}$ inch	37	.18
$\frac{1}{32}$ inch	16	.08

EXERCISES

1. A half-inch hose is used in watering the lawn. What is the cost (to the nearest cent) for water, if the hose is used for 15 minutes each day of July for this purpose? 22 days in August?

2. When the leak from a faucet is equivalent to a stream from a $\frac{1}{32}$ in. pipe, what is the cost of letting this faucet leak for a week? For a month?

3. The plumber charges 80 cents per hour and takes $\frac{3}{4}$ hr. to fix such a faucet as in Ex. 2. In how many days would the water wasted equal the bill for fixing the faucet?

4. A fountain is supplied by a $\frac{1}{8}$ in. pipe. What is the cost of water for this fountain for July and August?

5. A truck gardener uses a $\frac{1}{4}$ in. hose on an average of 45 min. a day from June 15 to Sept. 15. What is the cost of the water used?

6. An automatic sprinkler, attached to a half-inch hose, is allowed to run on an average of 2 hr. a day for 20 days in July and 18 days in August. What is the cost of the water used?

III. Arithmetic of Civic Life

§ 35. Taxes. A ***tax*** is a sum of money levied by the town, city, state, or national government for the payment of public expenses.

Taxes are of two kinds — ***direct*** and ***indirect.*** The most common direct taxes are ***property***, ***income***, and ***poll*** taxes. The indirect taxes are the ***duties*** on imports and the ***internal revenue*** charges on certain manufactured products.

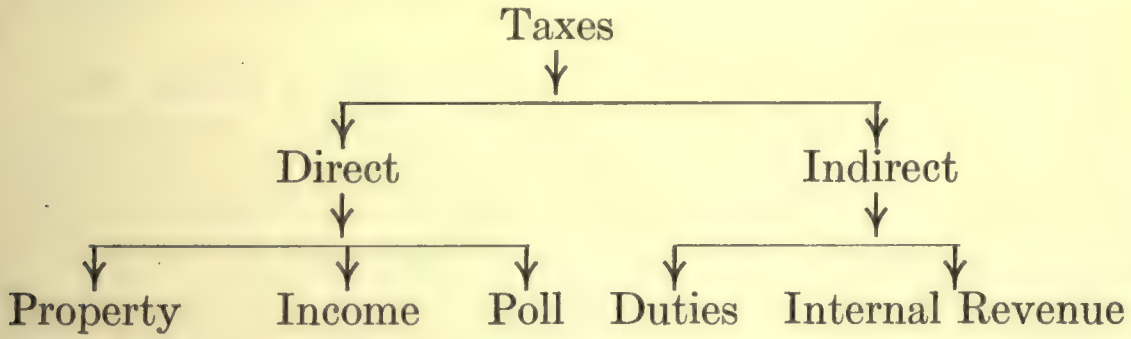
A ***property tax*** is a tax levied on the property, real estate and personal, which an individual owns.

Real estate is immovable property; such as, land, houses, factories, etc.

Personal property is movable property; such as, cattle, automobiles, pianos, stocks and bonds, etc.

An ***income tax*** is a tax levied on a person's income in excess of a certain sum.

A ***poll tax*** is a tax levied, in some states, on each male inhabitant over 21 years of age.



§ 36. Propert Tax. The ***assessed value*** of property is the value placed upon it for purposes of taxation by the proper officers (assessors). The assessed value of property is usually a fractional part of its real value; such as, $\frac{2}{3}$ or $\frac{3}{4}$.

The tax rate may be expressed in any one of the following ways:
the number of dollars on each \$1000 of *assessed valuation*,
or the number of cents on each \$100 of *assessed valuation*,
or the number of mills on each \$1 of *assessed valuation*,
or a certain per cent of the *assessed valuation*.

The tax rate for the city of Boston for 1916 was \$17.80 on \$1000 of assessed valuation. This might have been given as \$1.78 on \$100, or 17.8 mills on \$1, or 1.78%.

EXAMPLE. The assessed value of A's house in the city of Boston is \$6500. What was the amount of his tax on this house for 1916, if the tax rate for that year was \$17.80 per \$1000?

SOLUTION.

\$6500 at \$17.80 per \$1000 is

$6.5 \times \$17.80 = \$115.70$

Ans. \$115.70.

EXERCISES

Using the tax rate of \$17.80 per \$1000, find the taxes on each of the following:

PROPERTY	VALUATION	TAXES
1. Single house	\$2,500	?
2. Single house	3,500	?
3. Single house	4,200	?
4. Single house	6,250	?
5. Two-family house	7,500	?
6. Two-family house	8,300	?
7. Two-family house	8,550	?
8. Two-family house	9,500	?
9. Apartment house	20,500	?
10. Apartment house	112,500	?

11. I own a house and lot, the assessed value of which is \$5250, and an automobile valued at \$500. What is the total amount of my taxes when the tax rate is \$18.20 per \$1000? When it is \$18.90 per \$1000?

Find the amount of the tax on each of the following:

Property	Valuation	Tax Rate		Tax	
		1915	1916	1915	1916
12. Single house . .	\$5,250	\$18.20	\$18.90	?	?
13. Single house . .	6,500	16.70	17.10	?	?
14. Two-family house	8,550	18.20	18.90	?	?
15. Two-family house	9,200	16.70	17.10	?	?
16. Two-family house	10,500	17.20	17.50	?	?
17. Apartment house	65,000	18.20	18.90	?	?
18. Apartment house	155,000	17.20	17.50	?	?
19. Business block .	250,000	18.20	18.90	?	?
20. Business block .	265,000	16.70	17.10	?	?

§ 37. Insurance. ***Insurance*** is a contract by which a company agrees for a stated amount, called a ***premium***, to make up the loss which one may suffer from stated causes.

The ***policy*** is the written contract. The ***face*** of the policy is the sum mentioned in the contract.

The ***premium*** is the money paid for the insurance. The cost of insurance may be given as a certain per cent of the face of the policy, or it may be a stated amount per \$100.

There are several kinds of insurance; such as, fire insurance, accident insurance, health insurance, liability insurance, automobile insurance, tornado insurance, and life insurance. Only fire insurance and life insurance will be considered here.

§ 38. Fire Insurance. When a company insures my house for $4000 against loss by fire, the company agrees to make good my loss to the extent of $4000 in case the house is accidentally burned during the period named in the policy.

Insurance companies frequently issue a policy for 3 years for a premium equal to two and one half times the annual premium and for 5 years for four annual premiums. Hence a lower rate is obtained if the policy is written for a period of three or five years.

EXAMPLE 1. My house is insured for $4000 at an annual rate of .3 per cent. What is the face of the policy? What is the annual premium?

The face of the policy is $4000.

The premium is .3 per cent of $4000, or $12.00.

EXAMPLE 2. The estimated value of my house is $6000 (Ex. 1). For what part of its value is it insured? For what per cent of its value is it insured? (It is not customary to insure property for its full value.)

EXAMPLE 3. On my household goods valued at $1200 I carry an insurance for $\frac{2}{3}$ their value. The rate is $\frac{3}{4}$ per cent for a period of three years. What is the face of the policy? What is the premium?

$\frac{2}{3}$ of $1200 = $800 (face of policy)

$\frac{3}{4}$ per cent = $\frac{3}{4}$ of 1 per cent.

$\frac{3}{4}$ of $8.00 = $6.00 (premium)

EXAMPLE 4. A house valued at $7500 is insured for $\frac{2}{3}$ of its value at 75¢ per $100 for a period of three years. What is the premium?

$\frac{2}{3}$ of $7500 = $5000 (face of policy)

$50 \times \$0.75 = \$37.50$ (premium)

EXERCISES

In each of the following exercises, the face of the policy is $\frac{3}{4}$ of the estimated value of the property. Determine in each case the face of the policy and the premium.

	Property	Estimated Value	Rate	Face of Policy	Premium
1.	House	$4,000	$\frac{3}{4}\%$	$3000	$22.50
2.	House	6,000	$\frac{3}{4}\%$	?	?
3.	Garage	600	$2\frac{1}{4}\%$	?	?
4.	Garage	720	3%	?	?
5.	Schoolhouse	10,000	50¢ per $100	?	?
6.	Schoolhouse	16,000	75¢ per 100	?	?
7.	Stock of goods	2,400	$1.25 per 100	?	?
8.	Stock of goods	7,200	$1.50 per 100	?	?
9.	Factory	50,000	$1.35 per 100	?	?
10.	Factory	80,000	$2\frac{1}{4}\%$	?	?
11.	Church	36,000	$1.50 per $100	?	?
12.	Church	160,000	$1\frac{1}{2}\%$	?	?
13.	Apartment house	240,000	$1\frac{1}{4}\%$	?	?
14.	Factory	300,000	2%	?	?

15. The cost of insuring a house for $5000, for a term of three years, is $37.50. What is the rate for this period?

16. The cost of insuring my automobile for $600, for one year, is $7.50. What is the rate?

17. The cost of an insurance of $350 on a piano, for one year, is $8.75. What is the rate?

18. It costs the city $1125 to carry an insurance of $75,000, for one year, on a certain schoolhouse. What is the rate?

19. The annual rate of insurance on a factory is given as $1.75 per $100. What is the annual cost for an insurance of $75,000 on this factory?

20. A certain village carries an insurance for $\frac{3}{4}$ the value on the buildings of a State Normal School located there. The buildings are valued at $300,000. This insurance is distributed equally among three companies whose rates for a period of three years are as follows: Company A, $1.65 per $100; Company B, $1.70 per $100; Company C, $1.73 per $100. Find the amount of premium paid to each company for this insurance.

§ 39. Life Insurance. The three most common kinds of life insurance policies are described below:

	I	II	III
Comparative features	Ordinary Life	Limited Life	Endowment
Annual cost	Lowest	Medium	Highest
Number of premiums	Each year (during life of insured)	Limited number (as specified in contract)	Limited number (as specified in contract)
Maturity	At death of insured	At death of insured, or a cash value at the end of term of contract	At death of insured, or at the end of term of contract

The amount of the annual premium is usually stated at a rate per $1000. (See table.)

ANNUAL PREMIUMS, $1000 — UNIFORM PREMIUM CONTRACT

AGE	ORDINARY LIFE	20–PAYMENT LIFE	20–YEAR ENDOWMENT	30–YEAR ENDOWMENT	AGE
21	$15.56	$23.06	$41.88	$26.55	21
25	17.06	24.67	42.17	26.90	25
30	19.42	27.06	42.67	27.67	30
35	22.50	30.00	43.43	28.94	35
40	26.56	33.67	44.68	31.07	40

EXERCISES

1. At the age of 21, I insured my life for $1000 on the 20-Payment Life plan. Find from the table on page 94 the annual premium. If I have paid 17 annual premiums, how much has this insurance cost me?

2. At the age of 25, I took an insurance for $2000 on the 30-Year Endowment plan. Find the amount of the annual premium. If I have paid 13 annual premiums, how much has this insurance cost me?

3. I am 35 years old. What will it cost me, annually, to take out a policy for $5000 on the Ordinary Life plan?

4. What will it cost me, annually, to take out a policy for $3000 on the 20-Year Endowment plan, age 40 years?

5. A 20-Payment Life policy for $1000, taken at the age of 21, has a cash value at the end of the 20th year of $419. How much greater is the total of the 20 annual premiums paid?

6. The premium on an Ordinary Life policy for $1000, taken at the age of 25, if paid semiannually, is $8.87. How much is saved in a period of 10 years by paying the premiums on this policy annually? (Consult the table.)

7. The premium on the policy in Ex. 6, if paid quarterly, is $4.52. What is the cost of this insurance for 10 years, if the premiums are paid quarterly?

8. A 20-Payment Life policy for $2000, taken at the age of 30, on the Premium Reduction plan, costs $34.13 per $1000 for the first year and $25.45 per $1000 for each subsequent year. Find the total amount paid in premiums on this policy for 20 years. Compare the amount paid in premiums on this policy with what would have been paid on the uniform premium plan. (See the table.)

CHAPTER V

PLANE GEOMETRIC FIGURES

I. Lines and Angles

§ 40. Arcs of Circles. In the first year course you learned how to measure plane figures and how to draw many of these figures. From your measurements you learned certain facts about them. For this work you used a protractor and a ruler. For the further study of these figures you will use another instrument, the compasses. With it you can measure off distances and draw circles.

EXERCISES

1. Using the same point for the center draw two circles, one having a radius $\frac{3}{4}''$ and the other having a radius $1\frac{1}{4}''$. These circles are *concentric circles*, because they have the same center.

2. Draw two circles that cut each other, one having a radius $\frac{3}{4}''$ and the other having a radius $1\frac{1}{4}''$. These circles are *intersecting circles*, because they cut each other.

NOTE. The two circles intersect at *two* points.

3. Draw two intersecting circles that have the same radius. Is it necessary to draw the entire circles to get the points of intersection?

4. What is a circle? The radius of a circle? The circumference of a circle? An arc of a circle? The diameter of a circle?

§ 41. Perpendiculars and Parallels. The ***perpendicular bisector*** of a given line is the line that forms right angles with the given line at its middle point.

Parallel lines are lines in the same plane that do not meet however far they may be extended.

EXERCISES

In these exercises the word *line* will be used to mean *straight line.*

1. *To construct the perpendicular bisector of a given line.* Draw the line AB for the given line. (See Fig. 13.) About A as a center draw an arc, using a radius more than one half the length of AB. About B as a center draw an arc, using the same radius. Make the arcs long enough so that they will intersect in two points.

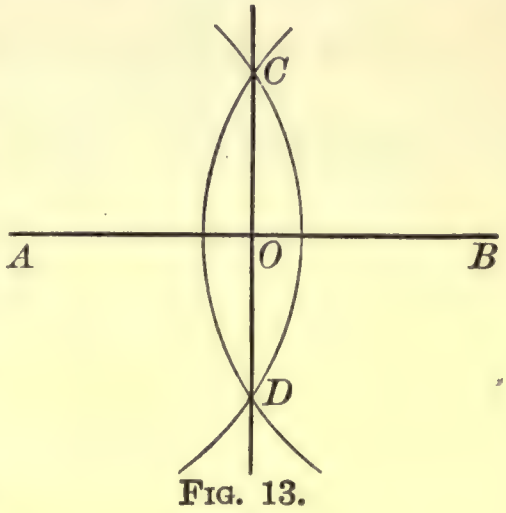


FIG. 13.

Mark the two points C and D. Draw a line through C and D. CD is the perpendicular bisector of AB, at O.

(*a*) Test the accuracy of your construction with your protractor and ruler.

(*b*) Why should the radius be more than one half the length of AB?

(*c*) Using A and B as centers, but this time with a longer radius, draw arcs that intersect. Do these points of intersection lie on the line CD extended?

(*d*) Compare the distances from any point in CD to the end points A and B.

NOTE. Every point that you choose in CD is equally distant from A and B.

2. *To construct a perpendicular to a given line through a given point.*

A. When the given point is on the line.

Draw the line AB for the given line. (See Fig. 14.) Mark a point P on AB for the given point.

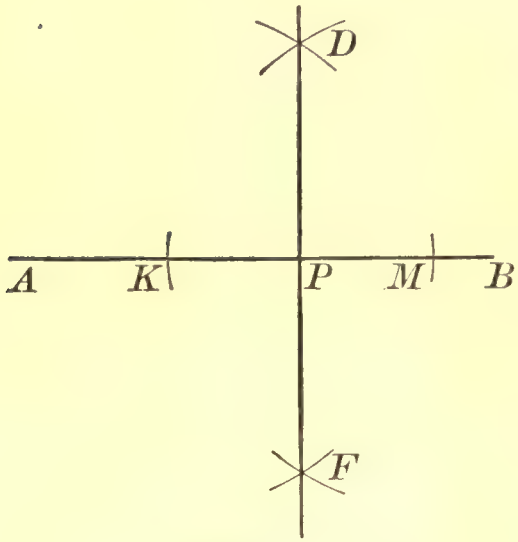


Fig. 14.

With the compasses measure off on AB from P equal distances, PK and PM.

Using K and M as centers and a radius more than one half the length of KM, draw arcs intersecting at D and F.

Draw DF. DF is the perpendicular to AB through P.

(*a*) Test the accuracy of your construction with your protractor.

(*b*) In this exercise, are both points of intersection D and F necessary for locating the perpendicular?

B. When the given point is not on the line.

Draw the line AB for the given line. (See Fig. 15.) Mark a point P *not* on AB for the given point.

Using P as a center and a radius sufficiently long, draw an arc cutting AB at X and Y.

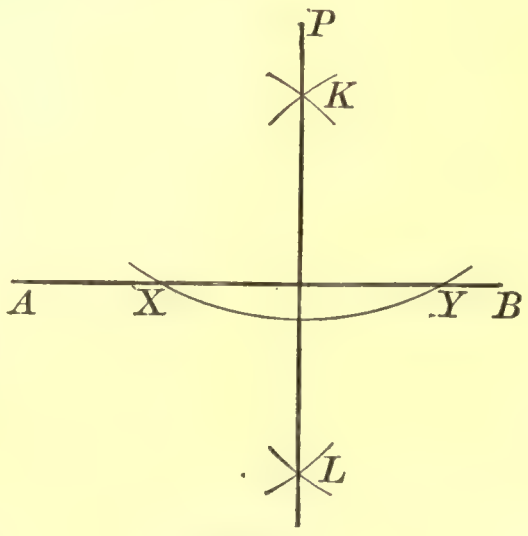


Fig. 15.

Using X and Y as centers and a radius more than one half the length of XY, draw arcs intersecting at K

and L. Draw a line through K and L and it will pass through P.

PL is the perpendicular to AB through P.

(a) Test it.

(b) Are both points K and L necessary?

(c) If one is to be left out, which one should it be? Why?

NOTE. Two points are sufficient to locate the direction of a straight line.

3. *To construct a line parallel to a given line through a given point.*

Draw the line AB for the given line. (See Fig. 16.)

Mark a point P *not* on AB for the given point.

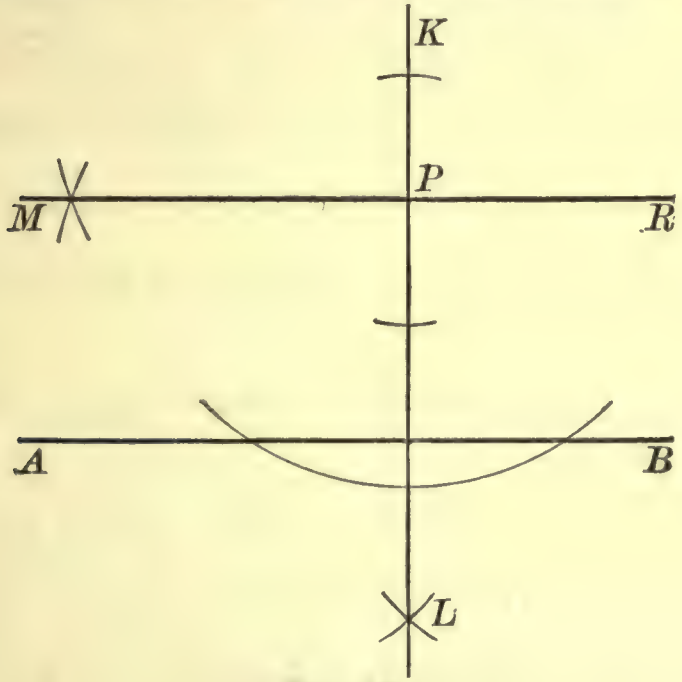


FIG. 16.

Through P construct KL perpendicular to AB, following the directions in Ex. 2 (B).

Through P construct MR perpendicular to KL, following the directions in Ex. 2 (A).

MR and AB are parallel lines, for *two lines perpendicular to the same line are parallel.*

§ **42. Angles.** An ***angle*** is the amount of rotation of a line about one of its points from one position to another. The ***bisector*** of an angle is the line which divides the angle into two equal parts.

EXERCISES

1. *To construct a given angle at a given point on a given line.*

Draw the angle A for the given angle. (See Fig. 17.) Draw CD for the given line and mark point P on CD for the given point.

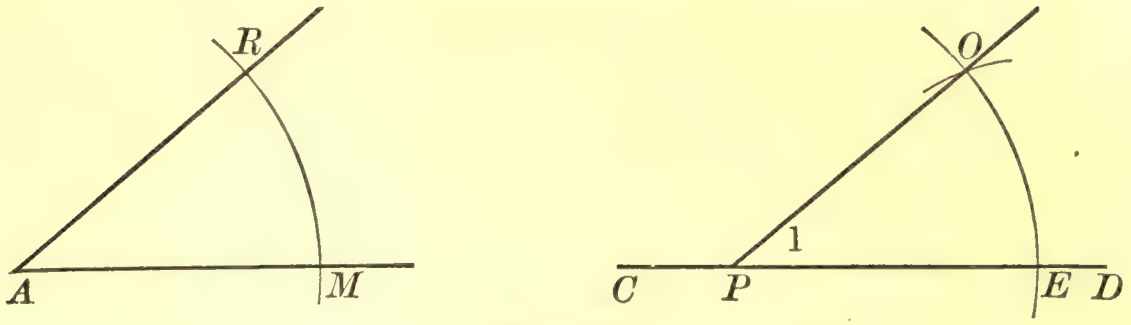


FIG. 17.

Using A as a center and a radius of convenient length, draw an arc cutting the sides of $\angle A$.

Using P as a center and the same radius, draw an arc cutting PD at E.

Using E as a center and a radius equal to MR, draw an arc cutting the arc whose center is P. Mark this point of intersection, O.

Draw a line through P and O forming $\angle 1$. $\angle 1$ is the same as $\angle A$ moved into a new position. Test the equality of $\angle 1$ and $\angle A$ with your protractor.

2. *To bisect a given angle.*

Draw $\angle ABC$ for the given angle. (See Fig. 18.)

Using B as a center and a radius of convenient length, draw an arc cutting the sides of the angle at R and S.

Using R and S as centers and a radius more than one half the length of RS, draw arcs intersecting at X.

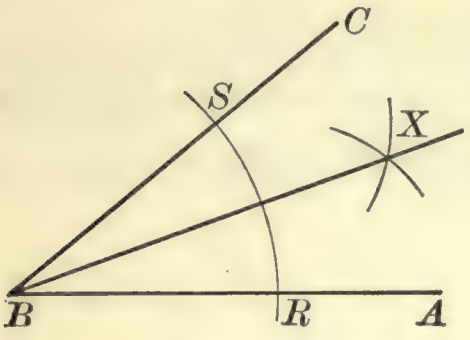


FIG. 18.

Draw a line through B and X.

BX bisects $\angle ABC$.

Test the equality of the two angles.

3. Draw an angle of 45°, using your compasses and ruler. [See Ex. 2 (A), page 98, and Ex. 2, page 100.]

4. Draw an angle of 135°. (See Fig. 19.)

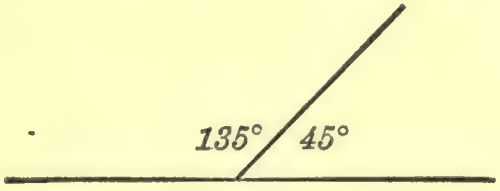


FIG. 19.

An angle of 135° is the *supplement* of an angle of 45°.

Supplementary angles are angles whose sum is 180°. One angle is the *supplement* of the other.

5. Draw an angle of 22° 30′; an angle of 167° 30′. Are these angles supplementary?

6. On a line AB, 2″ long, construct an angle of 45° at A and an angle of 90° at B. Continue the lines to form a triangle.

II. Triangles

§ 43. Classification. A ***triangle*** is a plane figure enclosed by three straight lines.

A. Triangles (△) are divided into three groups according to the length of their sides:

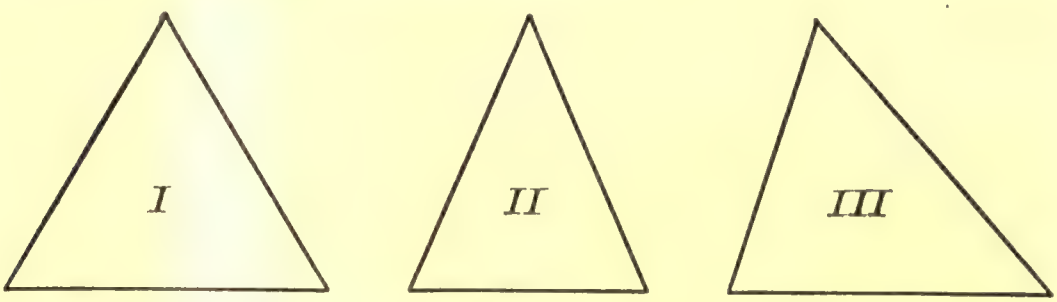


Fig. 20.

(*a*) *Equilateral* triangle, having all three sides equal.

(*b*) *Isosceles* triangle, having two sides equal.

(*c*) *Scalene* triangle, having no two sides equal.

B. Triangles are divided into three groups according to their angles:

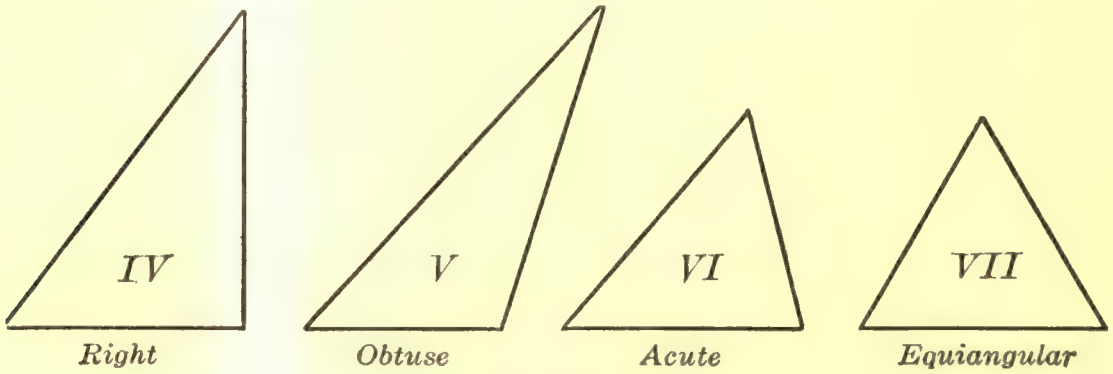


Fig. 21.

(*a*) *Right* triangle, having one angle right (90°).

(*b*) *Obtuse* triangle, having one angle obtuse (more than 90°).

(*c*) *Acute* triangle, having all three angles acute (less than 90°). It is *equiangular* if all three angles are equal.

EXERCISES

1. In Fig. 22, find all the triangles that you can and group them:

(*a*) According to the length of their sides, — as equilateral, isosceles, or scalene.

(*b*) According to their angles, — as right, obtuse, or acute.

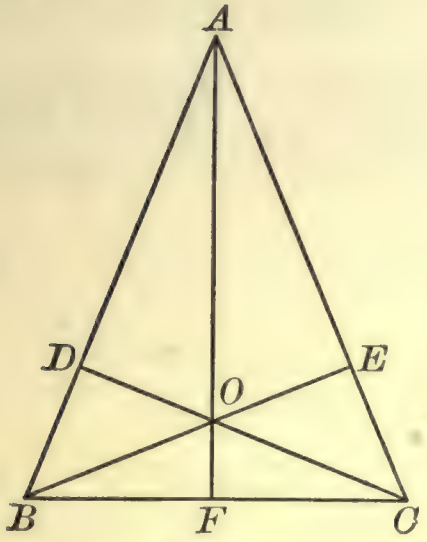


FIG. 22.

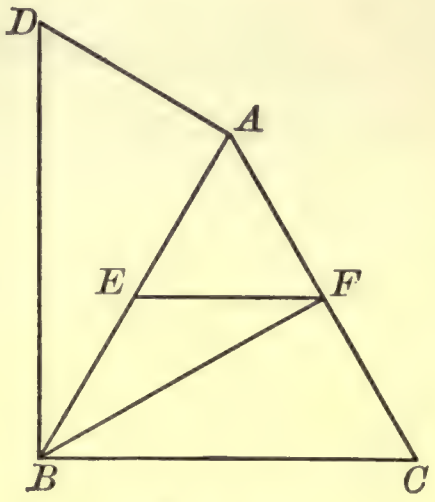


FIG. 23.

2. In Fig. 23, find all the triangles that you can and group them:

(*a*) According to the length of their sides.

(*b*) According to their angles.

3. With your protractor measure the angles of each of the triangles in Figs. 20 and 21, and fill in the required values in the following table.

	I	II	III	IV	V	VI	VII
∠1							
∠2							
∠3							
Sum							

(*a*) What seems to be true of all the angles of the equilateral triangle? How many degrees are there in each?

(*b*) What seems to be true of two of the angles of the isosceles triangle?

§ 44. Constructions.

EXERCISES

1. Construct an equilateral △ having each of its sides 1.5″ long.

Draw the line AB 1.5″ long (Fig. 24). Using A and B as centers and a radius 1.5″, draw arcs intersecting at C.

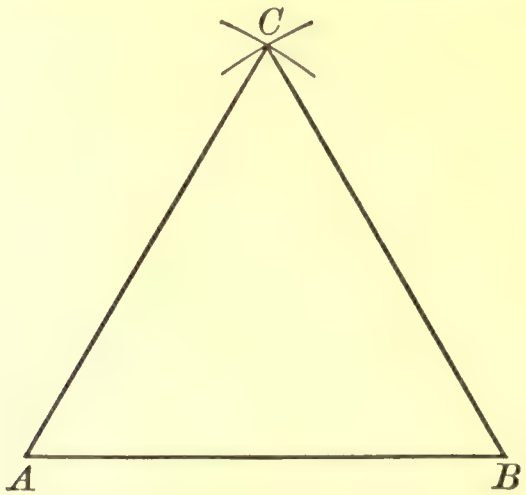


FIG. 24.

Draw AC and BC.

2. Construct an angle of 60°; an angle of 120°; an angle of 30°.

3. Construct an isosceles △, having its base 1.9″ and its equal sides each 2.25″.

(*a*) Measure the two angles at the base and compare them.

(*b*) Construct the altitude. Use the method given in Ex. 2 (*B*), page 98. The point from which the perpendicular is drawn is the meeting point of the two equal sides of the △. It is called the *vertex* of the triangle.

4. Construct a scalene △ whose sides are 2.5″, 1.25″, and 2″.

Lay off the longest side for the base.

With one end of the base as a center and 1.25″ for a radius, draw an arc.

With the other end as a center and 2″ for a radius, draw an arc cutting the first arc.

Complete the triangle.

5. Using the △ drawn for Ex. 4, construct the altitude of the △. (See Ex. 2 (*B*), page 98.)

6. Construct a △ *ABC*, having *AB* (the base) $2\frac{1}{2}''$, $\angle A = 45°$, and $AC = 1\frac{1}{4}''$.

7. Using the △ drawn for Ex. 6, construct a line bisecting $\angle C$. (See Ex. 2, page 100.)

8. Construct a △ *ABC*, having *AB* (the base) $2\frac{1}{2}''$, $\angle A = 45°$, and $\angle B = 60°$.

9. Using the △ drawn for Ex. 8, construct the perpendicular bisector of the base. (See Ex. 1, page 97.)

10. Draw any scalene △. Bisect the base. (See Ex. 1, page 97.) Join the middle point of the base with the opposite vertex. The line so drawn is a *median* of the △.

11. Draw a △ whose sides are 3″, 2″, and $1\frac{1}{2}''$. (Let the 3″ side be the base.)

(*a*) Construct its altitude.

(*b*) Construct the bisector of the angle opposite the base.

(*c*) Construct the perpendicular bisector of the base.

(*d*) Draw the median.

(*e*) Can you tell how these lines would appear, if the base was 3″, and the other two sides were of equal length?

12. In Fig. 25, ABC is an equilateral $\triangle$.

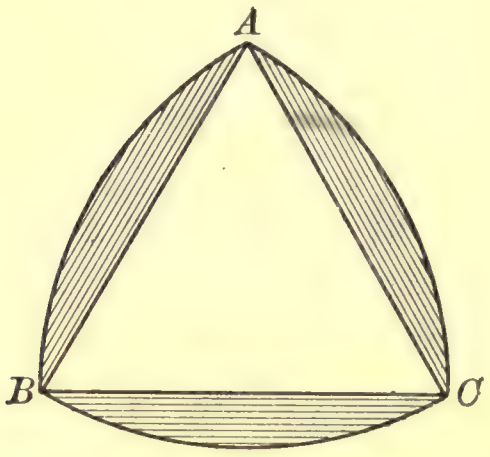


FIG. 25.

Draw the figure, letting $BC = 2.25''$.

NOTE. Figure 25 is drawn to one half size.

13. Construct Fig. 26.

(*a*) What kind of a triangle is ABC?

(*b*) What kind of a triangle is DEF?

(*c*) What are the other triangles?

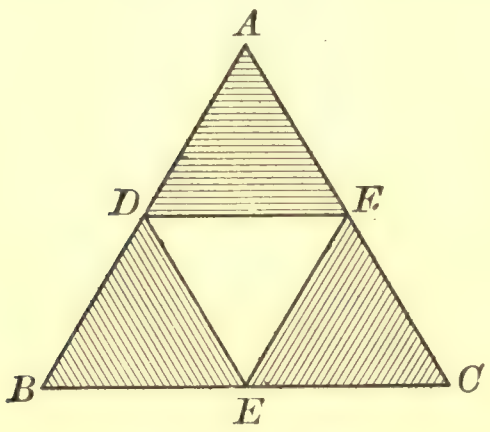


FIG. 26.

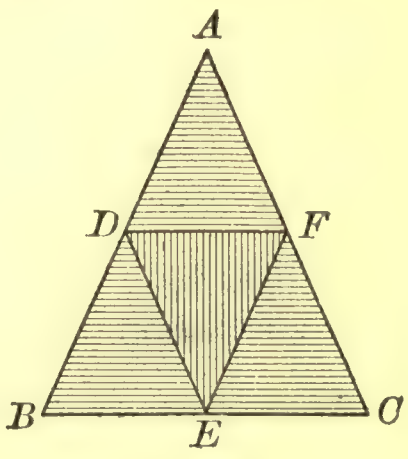


FIG. 27.

14. Construct Fig. 27.

(*a*) What kind of a triangle is ABC?

(*b*) What kind of a triangle is DEF?

(*c*) What are the other triangles?

III. Quadrilaterals and Other Polygons

§ 45. Classification of Quadrilaterals. — A ***quadrilateral*** is a plane figure inclosed by four straight lines.

A. *Quadrilaterals* are divided into three groups:

(*a*) *Parallelogram,* having opposite sides parallel.

(*b*) *Trapezoid,* having only two sides parallel.

(*c*) *Trapezium,* having no two sides parallel.

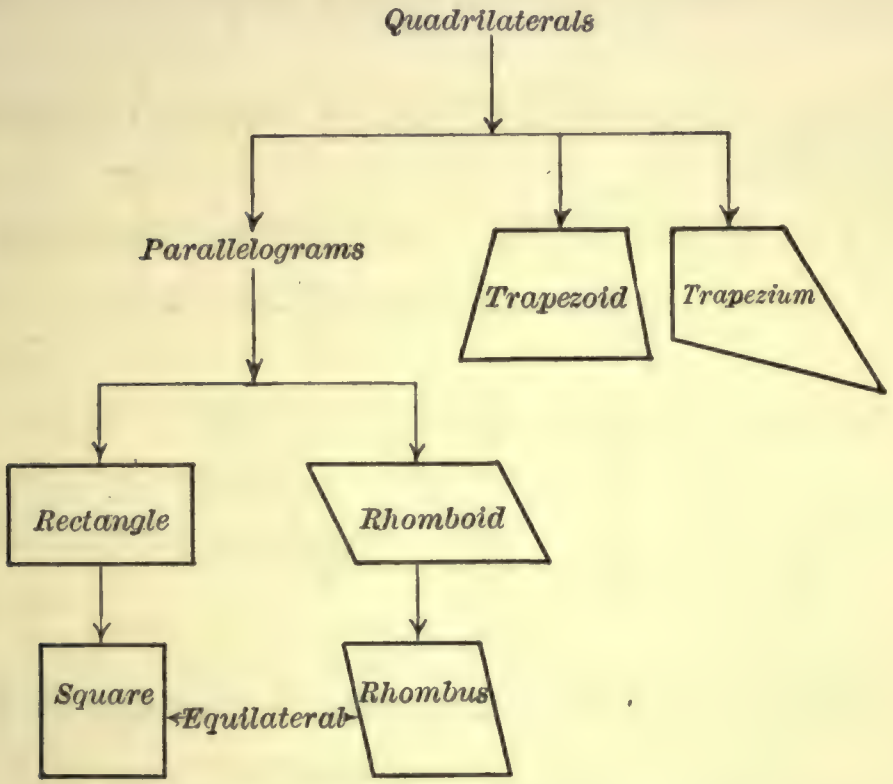


Fig. 28.

B. *Parallelograms* are divided into two groups:

(*a*) *Rectangle,* having all angles right angles.
A *square* is an equilateral rectangle.

(*b*) *Rhomboid,* having no angles right angles.
A *rhombus* is an equilateral rhomboid.

EXERCISES

1. In Fig. 29 find all the quadrilaterals that you can.

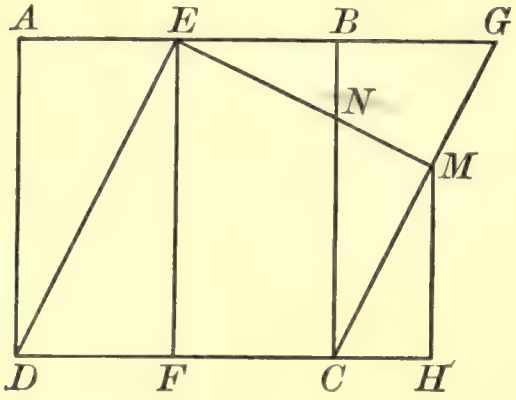


FIG. 29.

(*a*) Group them as parallelograms, trapezoids, or trapeziums.

(*b*) Group the parallelograms as squares, rectangles, or rhomboids.

2. In Fig. 30 find all the quadrilaterals that you can.

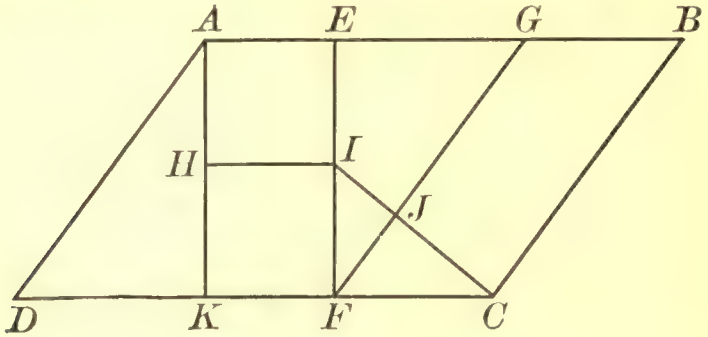


FIG. 30.

(*a*) Group them as parallelograms, trapezoids, or trapeziums.

(*b*) Group the parallelograms as squares, rectangles, or rhomboids.

§ 46. Constructions.

EXERCISES

1. Construct a parallelogram, if one angle is 60° and the sides including that angle are 2″ and 3″. (See Fig. 31, scale 2 to 1.)

C is the point of intersection of the two arcs. One of these arcs is formed by using D as a center and AB as a radius; the other is formed by using B as a center and AD as a radius.

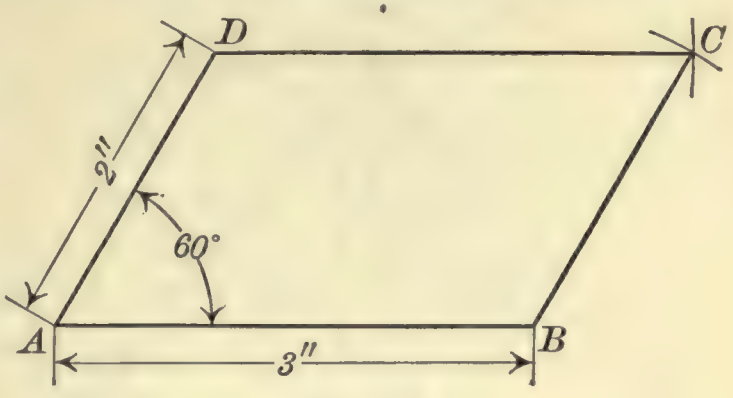


FIG. 31.

2. Construct a rhombus, having one of its angles 45° and its side $1\frac{3}{4}''$.

3. Construct a rhombus, having one of its angles 30° and its side 1.5″.

(*a*) Draw the two diagonals (lines joining the opposite vertices). With your protractor, measure the angles that they form with each other.

(*b*) Compare the lengths of the two parts of each diagonal.

(*c*) What is the ratio of the length of one diagonal to that of the other?

4. Construct a square, having each side 2″. Draw the diagonals.

(*a*) What kind of angles do the two diagonals make with each other?

(*b*) Compare the lengths of the two diagonals.

(*c*) What is the ratio of the length of the diagonal to the length of one side?

5. Construct a square, having each side 4″. Draw one diagonal.

(*a*) What is the ratio of the length of the diagonal to the length of one side?

(*b*) Compare the ratios obtained in Exs. 4 and 5.

6. Figure 32 represents a *kite* drawn to the scale of 1 foot to 1 inch.

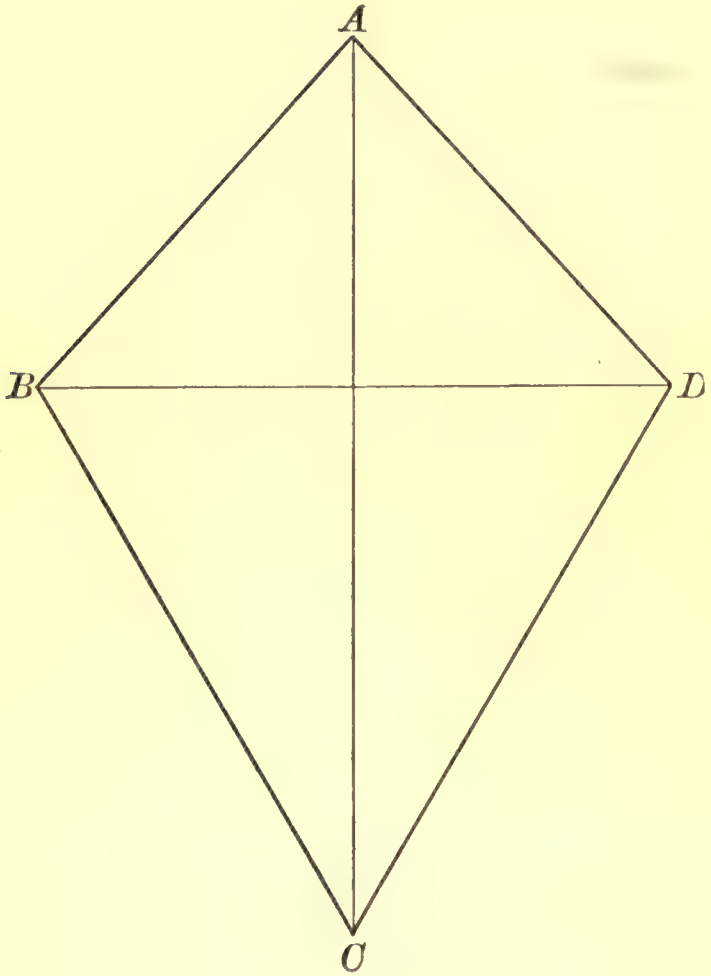


FIG. 32.

(*a*) Make a copy of the figure.

Suggestions. $AD = AB$, $CD = CB$; BD is 2 ft., AD is 1.5 ft., and CD is 2 ft.

(*b*) With your protractor and ruler, test your figure for the following facts:

(1) Every point in AC is equidistant from B and D.

(2) The axis (AC) of the kite is the perpendicular bisector of the other diagonal.

(3) The axis of the kite bisects the angles A and C.

NOTE. The kite $ABCD$ is said to be *symmetric* with respect to its axis, AC.

§ 47. Classification of Polygons. A ***polygon*** is a plane figure inclosed by three or more straight lines. These lines are the *sides* of the polygon; the points where the sides meet are the *vertices*.

A ***regular polygon*** is a polygon having all its sides equal and all its angles equal.

Regular polygons are generally named according to the number of sides. (See Fig. 33.)

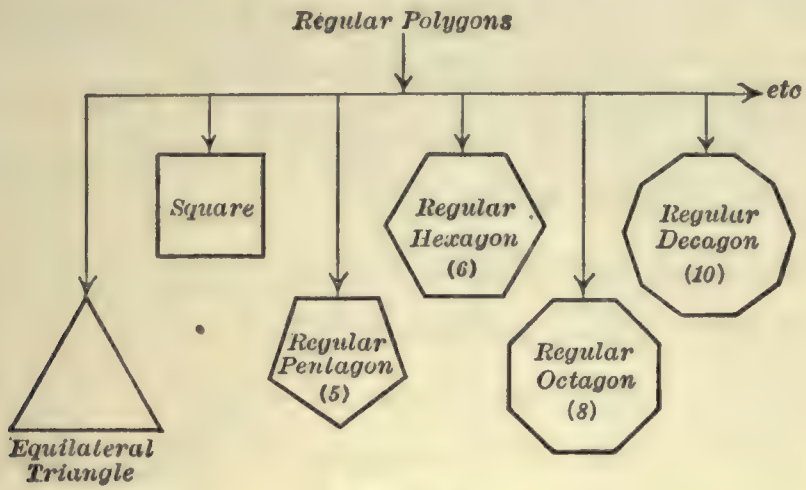


FIG. 33.

In the regular polygon $ABCDE$ (Fig. 34), O is the *center;* OA, OB, OC, OD, OE (radii) are equal. Also the five angles at O are equal. They are called the *angles at the center* of the polygon.

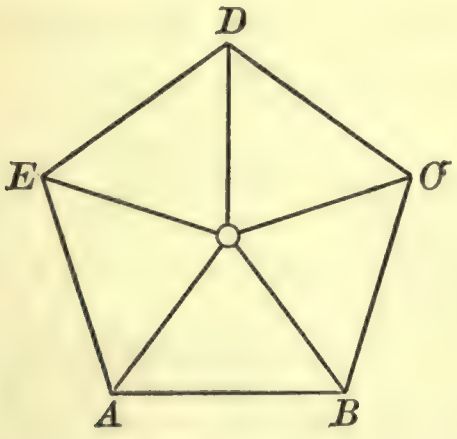


FIG. 34.

The angle at the center of any regular polygon $= \frac{360°}{n}$, *where n is the number of sides.*

In the case of a regular pentagon each of these angles $= \frac{360°}{5}$ or 72°. Measure each of the angles in Fig. 34.

§ 48. Constructions.

EXERCISES

Find the angle at the center and, with your protractor, construct the following regular polygons:

1. Pentagon, whose radius is $1\frac{1}{4}''$.
2. Square, whose radius is $\frac{3}{4}''$.
3. Octagon, whose radius is $1''$.
4. Decagon, whose radius is $1\frac{1}{4}''$.
5. Hexagon, whose radius is $2''$.

(*a*) Measure each of the angles in one of the triangles formed in this hexagon. How many degrees are there in each angle?

(*b*) Measure each of the three sides of one of the △. What kind of a triangle is it?

(*c*) Could you construct a regular hexagon, if the length of one side was given?

NOTE. The length of one side of the hexagon equals the length of the radius of the circle.

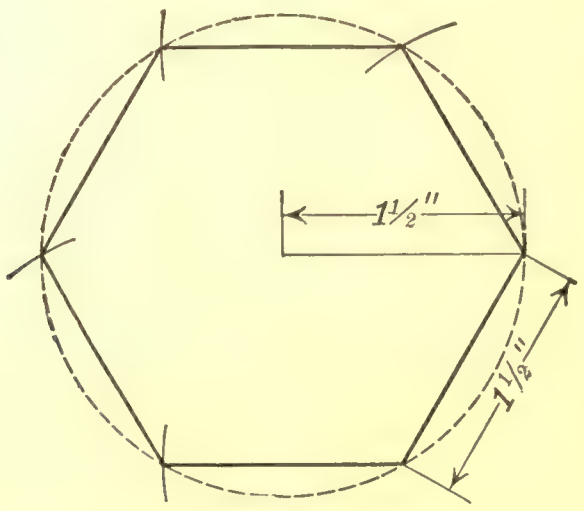


FIG. 35.

6. Construct a regular hexagon, if one side is $1\frac{1}{2}''$. (See Fig. 35.)

7. In Fig. 36, A, B, C, D, E, and F are the vertices of a regular hexagon, and O is its center.

(a) Construct this diagram by using the vertices of the hexagon as centers and OA for a radius.

(b) After you have made the construction, erase all lines that are dotted in Fig. 36, if you have drawn them in your figure.

(c) Shade or color the figure that remains.

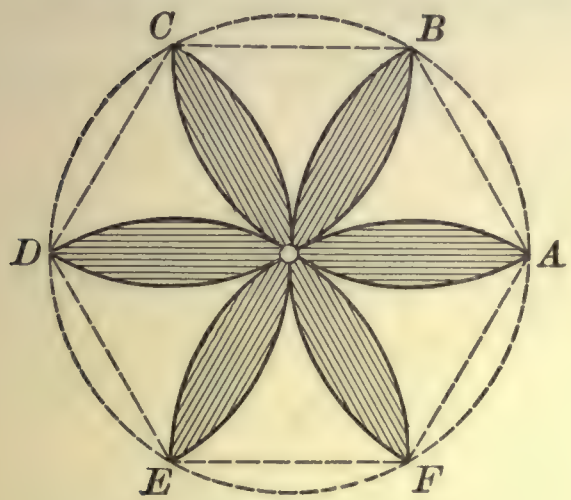


FIG. 36.

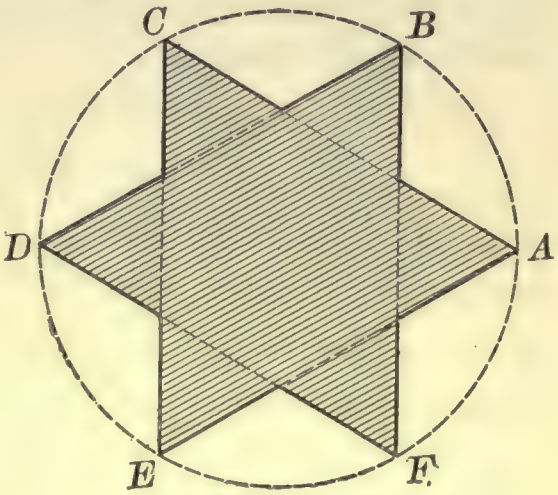


FIG. 37.

8. In Fig. 37, locate the points of the star as in Ex. 6. Join A with C, C with E, and E with A. Then join the other vertices in the same way. Erase the dotted lines in Fig. 37, if you have drawn them in your figure.

9. Construct a five-pointed star.

Suggestions. Construct a regular pentagon as in Ex. 1. Letter the vertices A, B, C, D, and E, in order. Join A with C, C with E, E with B, B with D, and D with A. Erase all lines but the star outline. Shade or color the star.

IV. Graphs of Number Data

§ 49. Line-Graphs.

In the following exercises, line-measurements are used to represent the amounts of other quantities for the purpose of comparing those quantities.

EXERCISES

1. The following line-graphs represent the comparative costs (in cents) of producing a given amount of light (1000 candle hours).

1 inch represents 10 cents.

Kerosene flame: ———————————————.

Gas open flame: ———————————————

Gas mantle: ————

"Gem" electric: ———————————————————————

Tungsten electric: ———————————————

Fig. 38

Measure each to the nearest tenth of an inch, and find the cost of each per 1000 candle hours.

Kerosene flame: 2.0″ long. $2.0 \times 10 = 20¢$, etc.

2. Represent by line-graphs the values of each of the following foreign coins, letting 1 inch represent 20 cents.

Great Britain:	shilling, 24.3¢
France:	franc, 19.3¢
Germany:	mark, 23.8¢
Japan:	yen, 49.9¢
Russia:	ruble, 51.5¢
Spain:	peseta, 19.3¢

3. The following data give, in square miles, the area of each of the Great Lakes. Represent these areas by line-graphs, letting 1 inch represent 10,000 sq. mi.

Lake Superior:	32,060 sq. mi.
Lake Michigan:	22,300 sq. mi.
Lake Huron:	23,000 sq. mi.
Lake Erie:	10,000 sq. mi.
Lake Ontario:	7,200 sq. mi.

§ 50. Circular Graphs.

In the exercise that follows the amounts of the quantities are represented by parts of a circle. Note that the circular graph is used only when you have the total of the groups to be represented.

EXERCISE

The population of the world in 1910 was about 1,700,000,000. The circular graph (Fig. 39) shows the division

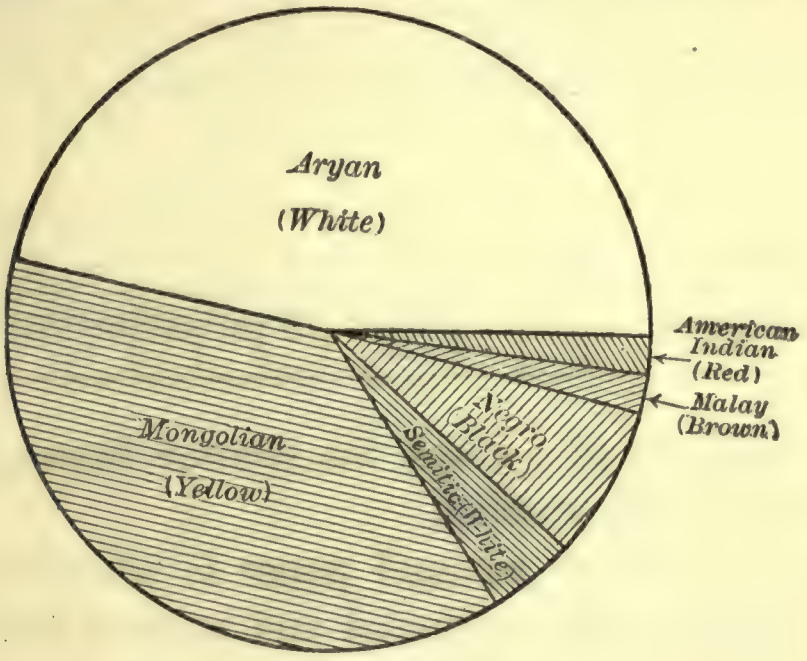


FIG. 39.

by races. Using your protractor, find how many degrees of the circle represent each race. Find in round numbers how many there are of each race.

Aryan (white) = $\frac{168}{360}$ of 1,700,000,000, or about 794,000,000; etc.

§ 51. Graphs on Squared Paper.

When the corresponding changes in two related quantities [for example, temperatures at successive hours of the day, or populations for successive years, etc.] are to be shown, it is customary to represent these by lines and points. The points locate the temperatures at certain hours, or the populations for certain years. The line joining these points in succession presents in a clear way the changes to be represented.

EXERCISES

1. The temperatures for each hour on a certain day were as follows:

Hour A.M.	12	1	2	3	4	5	6	7	8	9	10	11	12
Temperatures	53°	50°	48°	46°	43°	45°	49°	51°	54°	56°	60°	64°	72°

Hour P.M. . .	1	2	3	4	5	6	7	8	9	10	11	12
Temperatures .	75°	77°	77°	75°	73°	70°	68°	64°	63°	62°	62°	63°

Show by means of a graph the relation between the two quantities in this problem.

For this purpose squared paper is convenient. This paper is ruled with horizontal and vertical lines, equally spaced, which divide the paper into small squares, usually one *tenth* of an inch long on each side. In this diagram (Fig. 40) you will note that the horizontal lines represent the *time*, each unit representing one hour; the vertical line represents the *temperature*, each unit representing one degree, beginning at 40°. The first point is located at

53° directly over 12, the second point at 50° directly over 1, and the next at 48° directly over 2, etc. Explain the location of the other points.

This figure does not represent the temperature between the hours, for that is not accurately known from the data.

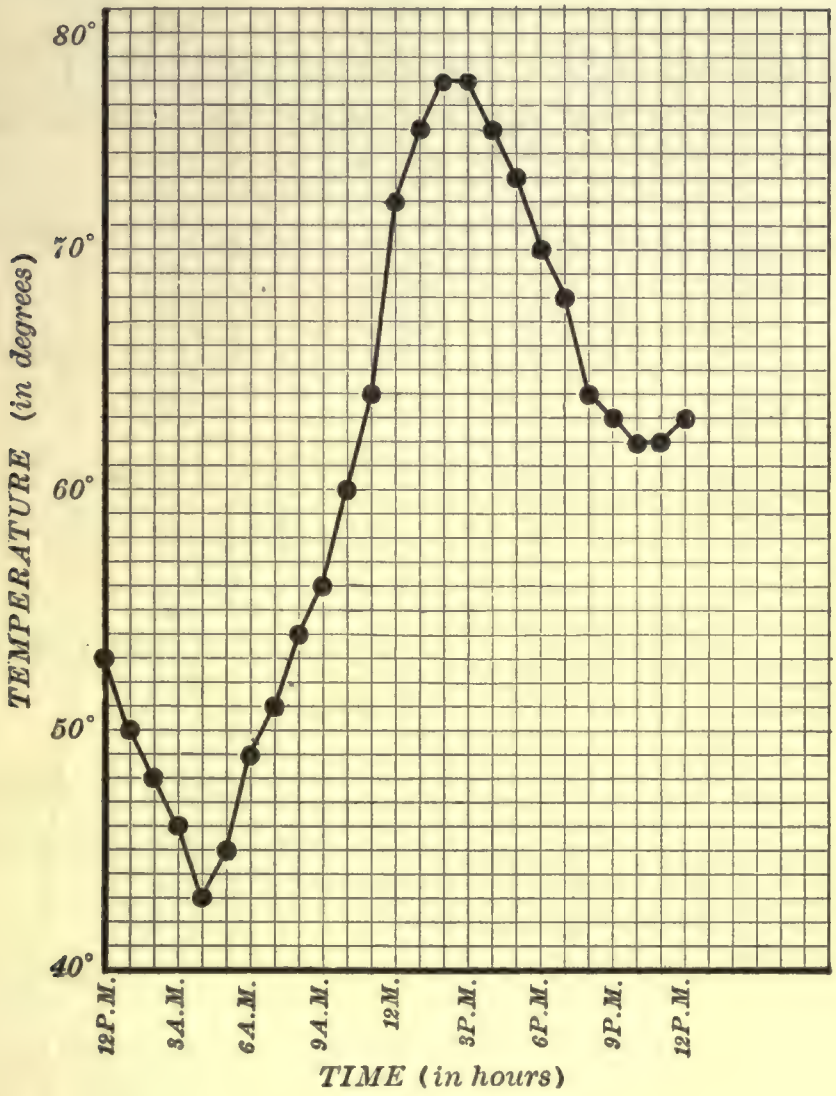


FIG. 40.

Noticing that the temperature changes gradually, if we draw a continuous line through these points, we can tell several things about the temperature. It dropped gradually during the early morning and rose rapidly during the forenoon. It reached the greatest height at 2 P.M. and remained stationary for an hour. It dropped some

toward evening, but rose again toward midnight, indicating the probability of warmer weather on the next day.

NOTE. This work is called *plotting* the *graph* for the temperatures given in the table.

(*a*) What probably happened between 10 and 12 P.M.?

(*b*) What was the probable temperature at 8.30 A.M.?

(*c*) At about what time was the temperature 74°?

(*d*) What do you find printed along the vertical line, or axis?

(*e*) What do you find printed along the horizontal axis?

Be sure to label the axes of each graph to agree with the data given.

NOTE. Instruments are used at the weather bureaus which record automatically on squared paper the graphs of temperatures, of rainfall and of air pressure, so that those in charge of the bureaus do not have to keep watch and record the changes.

2. The following table gives the number of deaths in the United States from 1905 to 1916 due to Fourth of July accidents.

YEAR	NUMBER	YEAR	NUMBER
1905	182	1911	57
1906	158	1912	41
1907	164	1913	32
1908	163	1914	40
1909	215	1915	30
1910	131	1916	30

Plot a continuous graph to represent these data. Let 1 inch on the vertical axis represent 50 people and let 1 inch

on the horizontal axis represent 5 years as in Fig. 41. In this figure the points for 1905 and 1906 are located.

(*a*) What is the probable cause of the large drop in the graph?

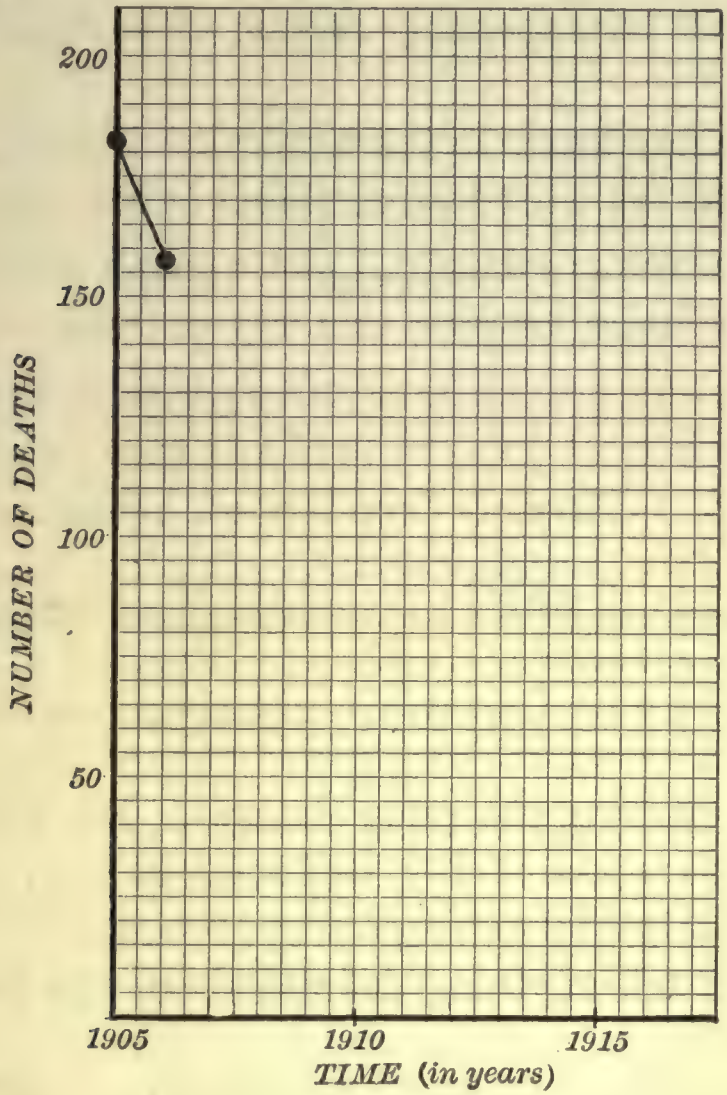


FIG. 41.

(*b*) Be sure to label the axis and locate the numbers as given in Fig. 41. (Fig. 41 is $\frac{3}{4}$ size.)

3. The population of the United States, in millions, since 1800 is given in the census reports as follows:

1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1915
4.3	7.2	9.6	12.9	17.1	23.2	31.4	38.6	50.2	62.6	76.3	92.0	100

Plot the graph showing the growth in population. Let 1 inch on the vertical axis represent 20 million, and let 1 inch on the horizontal axis represent 50 years.

(*a*) From the graph estimate the probable population in 1920. Can you be reasonably certain of your estimate? Why?

(*b*) Is there any break in the regularity of your graph? If so, can you tell what historical fact is the cause of it?

4. Get the necessary data and plot a graph showing the change in population for your own city or town.

5. The following are the farm prices per acre of crops for five-year periods from 1870 to 1910 and then by one-year periods to 1916. (Monthly Crop Report, February, 1917.)

1870	1875	1880	1885	1890	1895	1900	1905	1910	1911	1912	1913	1914	1915	1916
118¢	98¢	103¢	84¢	102¢	67¢	82¢	94¢	116¢	126¢	108¢	128¢	116¢	116¢	178¢

Plot a graph showing the price changes. Let 1 inch on the vertical axis represent 20¢, beginning at 90¢; and let 1 inch on the horizontal axis represent 10 years.

(*a*) Can you tell anything definite about the probable price for 1920?

6. From the American Experience Table of Mortality used by insurance companies, the table at the top of page 121 shows the average number of years that people live after reaching the age given. Plot the graph. (Vertical axis: 1″ for 10 years to live; horizontal axis: 1″ for 25 years.)

(*a*) From the graph find out how many years a 13-year-old boy is expected to live.

(*b*) How many years is a 32-year-old man expected to live?

Age (In Years)	Years to Live	Age (In Years)	Years to Live
10	48.7	55	17.4
15	45.5	60	14.1
20	42.2	65	11.1
25	38.8	70	8.5
30	35.3	75	6.3
35	31.8	80	4.4
40	28.2	85	2.8
45	24.6	90	1.4
50	20.9	95	.5

7. The following data were taken from the report of the Massachusetts State Board of Education (1916).

A. The average weekly earnings of high school graduates are the following:

Age	18 Years	20 Years	23 Years	25 Years
Earnings	$10	$15	$21	$31

B. The average weekly earnings of those leaving school at the end of the eighth grade are the following:

Age	14 Years	18 Years	20 Years	23 Years	25 Years
Earnings	$4	$7	$9.50	$11.85	$12.75

On the same sheet of squared paper using the same axes, plot the graphs for these two sets of data. Let 1 inch on the vertical axis represent 5 years, and let 1 inch on the horizontal axis represent $10.

Note. The graph for (*A*) starts at 18 yr., and the graph for (*B*) starts at 14 yr.

8. A cubic foot of water weighs 62.5 lb. The weight of water may be expressed by the formula:

$$W = 62.5\ V,$$

where W = the weight, or quantity, of water in pounds,

and V = the volume in cubic feet.

Tabulate a set of values as follows:

V (in cubic feet) . .	1	2	4	6	8	10
W (in pounds) . . .	62.5	125	250	375	500	625

Plot the graphs for these sets of values, 1″ along the horizontal axis representing 5 cu. ft., and 1″ along the vertical axis representing 200 lb.

NOTE. Start the graph from where the axes cross.

(*a*) From the graph find the weight of 5 cu. ft. of water. Of $7\frac{1}{2}$ cu. ft.

(*b*) From the graph find how many cubic feet there are in 200 lb. of water. In 300 lb.

9. The average rate of an aeroplane is 75 mi. per hr. The distance traveled may be expressed by the formula,

$$d = 75\,t,$$

where d = the distance traveled in miles,

and t = the time in hours.

As in Ex. 8, tabulate values for d corresponding to the values 2, 4, 6, 8, 10 for t. Plot the graph. Read from the graph pairs of corresponding values of d and t.

CHAPTER VI

FORMULAS — MENSURATION OF PLANE FIGURES

I. Simple Equations

§ **52.** An ***equation*** is the statement of equality of two number expressions. In Chapter II, First Course, equations were solved by one of the two methods following:

(*a*) Divide each member by the same number, using the axiom: *If equal numbers are divided by the same number, the quotients are equal.*

(*b*) Multiply each member by the same number, using the axiom: *If equal numbers are multiplied by the same number, the products are equal.*

Example 1. [Method (*a*).] Solve the equation

$$4\,m = 24,$$

and check the answer.

Solution.

① $\quad 4\,m = 24$

② $\quad m = 6 \qquad$ ① ÷ 4

(The symbol, ① ÷ 4, means that each member of equation ① is divided by 4.)

Check.

$$4 \times 6 = 24$$

Ans. $m = 6$.

Example 2. [Method (*b*).] Solve the equation

$$\frac{a}{3} = \frac{5}{2},$$

and check the answer.

SOLUTION.

①	$\frac{a}{3} = \frac{5}{2}$	
②	$2\,a = 15$	① $\times$ 6
③	$a = 7\frac{1}{2}$	② $\div$ 2

(The symbol, ① $\times$ 6, means that each member of equation ① is multiplied by 6.)

CHECK. $\frac{7\frac{1}{2}}{3} \stackrel{?}{=} \frac{5}{2}$

$2.5 = 2.5$

Ans. $a = 7\frac{1}{2}$.

EXERCISES

Solve each of the following equations, choosing the necessary method, and check each answer.

1. $5\,a = 20$
2. $4\,x = 10$
3. $6\,m = 15$
4. $3\,h = 24$
5. $2\frac{1}{2}\,a = 5$
6. $3.5\,x = 10.5$
7. $12\,x = 4$
8. $\frac{m}{6} = 5$
9. $\frac{y}{5} = \frac{3}{10}$
10. $\frac{1}{2} = \frac{x}{8}$
11. $\frac{3}{4} = \frac{n}{100}$
12. $15 = 12\,a$
13. $\frac{h}{4} = 3$
14. $8 = 5\,m$
15. $2 = \frac{b}{3}$
16. $12\frac{1}{2} = 2.5\,n$
17. $3\frac{3}{4} = \frac{n}{4}$
18. $5\frac{2}{3}\,c = 17$
19. $\frac{5}{8} = \frac{p}{100}$
20. $\frac{3\,a}{5} = 6$
21. $3.2\,a = 64$
22. $5\frac{1}{4} = \frac{n}{16}$

23. Solve the equation $7.2\,a = 11.8$, getting the value of a to the nearest third figure. Check the answer.

SOLUTION. To find the *nearest* third figure, it is necessary to know whether the fourth figure is less than 5 or more than 5.

①	$7.2\,a = 11.8$	
②	$a = 1.638$	① ÷ 7.2

The fourth figure is more than 5, so the value of a to the *nearest third* figure is 1.64.

CHECK.

$$7.2 \times 1.64 \stackrel{?}{=} 11.8$$
$$11.808 = 11.8$$

If the two members of the check agree to *three* figures, you may assume that the value of a is correct to the nearest third figure.

Ans. $a = 1.64$.

Solve each of the following equations, getting the values to the nearest third figure, and check each answer.

24. $3.7\,m = 8$

25. $2.6\,a = 15$

26. $19\,y = 78$

27. $.92 = 14\,k$

28. $378 = 144\,m$

29. $7.5\,x = 22$

30. $56 = 115\,p$

31. $3.5\,m = 18.6$

32. $22.4 = 3.15\,y$

33. $\frac{h}{3} = \frac{7}{11}$

34. $16.3 = \frac{11\,x}{4}$

35. $\frac{3\,x}{7} = 3.5$

36. $\frac{5\,y}{3} = 7\frac{1}{4}$

37. $17\frac{1}{3} = \frac{7\,m}{8}$

38. $9.3\,x = 11\frac{1}{2}$

II. Rectangles

§ 53. In Chapter V, First Course, some of the rules of arithmetic were expressed briefly as equations. These equations were called *formulas.*

Example 1. Write the formula for the statement:

The perimeter of a rectangle is equal to the sum of twice the base and twice the height.

Solution.

$$p = 2\,b + 2\,h,$$

where

p = the perimeter of a rectangle (in linear units),

b = the base (in linear units),

h = the height (in linear units).

When writing formulas for rules, each letter used in the formula should be fully explained, as in the model.

Example 2. $b = 17''$, $h = 12\frac{1}{4}''$, $p = ?$

Solution.

Placing 17 for b and $12\frac{1}{4}$ for h, in the formula,

$$p = 2\,b + 2\,h,$$

① $p = 34 + 24\frac{1}{2}$. (Estimate = 60″)

② $p = 58\frac{1}{2}$

① ≡

Ans. $p = 58\frac{1}{2}''$.

[The symbol, ① ≡, means that the members of equations ① and ② have *identical* values.]

You should make the statement of your answer complete by naming the unit used; that is, $p = 58\frac{1}{2}''$, not $p = 58\frac{1}{2}$.

EXERCISES

For Exs. 1–7 use the formula of Ex. 1.

1. $b = 5''$, $h = 4\frac{1}{2}''$, $p = ?$
2. $b = 3\frac{1}{4}''$, $h = 2\frac{1}{8}''$, $p = ?$

3. $b = 12'4''$, $h = 10'8''$, $p = ?$

4. $b = 12.2''$, $h = 6.3''$, $p = ?$

5. $b = 4.35''$, $h = 2''$, $p = ?$

6. Find the length of a fence inclosing a rectangular field 122′ long and 83′ wide.

7. The outside dimensions of a tennis court are 78′ and 36′. Find the length of the outside tape.

8. Write the formula for the statement:

The perimeter of a square is equal to four times one side. (Use the letters p and s.)

For Exs. 9–20, use the formula of Ex. 8.

9. $s = 3\frac{1}{2}''$, $p = ?$

10. $s = 7.4''$, $p = ?$

11. $s = 6\frac{3}{8}''$, $p = ?$

12. Find the length of a fence surrounding a square field 118′ long.

13. Find the distance around a baseball diamond. The distance from first base to second base is 90′.

14. $p = 20''$, $s = ?$

SOLUTION.

Placing 20 for p, in the formula,

①	$20 = 4\,s$	
②	$5 = s$	① ÷ 4

CHECK. $20 = 4 \times 5$

Ans. $s = 5''$.

15. $p = 140''$, $s = ?$

16. $p = 16.4''$, $s = ?$

17. $p = 14\frac{1}{2}''$, $s = ?$

18. $p = 45\frac{3}{4}'$, $s = ?$

19. $p = 3'\ 4''$, $s = ?$

20. The perimeter of a square picture frame is 37.2″. How wide is it?

21. Write the formula for the statement:

The area of a rectangle is equal to the product of its base and height. (Use A, b and h.)

For Exs. 22–39 use formula of Ex. 21. Write down an estimate for each result before computing it.

22. $b = 9''$, $h = 6''$, $A = ?$

NOTE. The *area* of a rectangle means the number of *square units* in the rectangle.

23. $b = 5\frac{1}{2}'$, $h = 4'$, $A = ?$

24. $b = 5.4''$, $h = 6.5''$, $A = ?$

25. $b = 3\frac{1}{2}'$, $h = 2'\ 6''$, $A = ?$

26. $b = 2'\ 6''$, $h = 1'6''$, $A = ?$

27. $A = 60$ sq. in., $b = 12''$, $h = ?$

SOLUTION.

Placing 60 for A and 12 for b,

①	$60 = 12\,h$	(Estimate $= 5''$)
②	$5 = h$	① $\div$ 12
CHECK.	$60 = 12 \times 5$	*Ans.* $h = 5''$.

Following the model in Ex. 27, find the required values in the following table. Copy the table and fill in the values.

FORMULA: $A = bh$.

	A		EST.	EQUATION	?
28.	50 sq. in.	$b = 10''$	$5''$	$50 = 10\,h$	$h = 5''$
29.	90 sq. ft.	$h = 7\frac{1}{2}'$	$\frac{100}{8}$ or $12'$	$90 = 7\frac{1}{2}\,b$	$b = 12'$
30.	150 sq. in.	$b = 15''$			$h = ?$
31.	6.5 sq. in.	$h = 5''$			$b = ?$
32.	$12\frac{1}{2}$ sq. ft.	$b = 2\frac{1}{2}'$			$h = ?$
33.	225 sq. in.	$b = 1'\ 3''$			$h = ?$
34.	18.9 sq. in.	$b = 3.5''$			$h = ?$
35.	$18\frac{3}{8}$ sq. ft.	$h = 3\frac{1}{2}'$			$b = ?$

36. Find the length of a gymnasium floor 45 feet wide and containing 4050 sq. ft.

37. A football field contains 52,800 sq. ft. and is 330′ long. How wide is it?

38. A lot of land contains 2400 sq. ft. and its length is 36′. Find how long a fence must be to inclose it.

(Use formulas for Exs. 21 and 1.)

39. Each of two lots of land contains 1800 sq. ft. One is 40′ long and the other 30′ long.

(*a*) Which has the longer perimeter?

(*b*) How many feet longer?

(*c*) What per cent longer?

40. $A = bh$. $b =$ what in terms of A and h?

SOLUTION. ① $A = bh$

② $\frac{A}{h} = \frac{bh}{h}$, or $\frac{A}{h} = b$ ① ÷ h

Ans. $b = \frac{A}{h}$.

41. $A = bh$. $h =$ what in terms of A and b?

42. Measure each of the lines in Fig. 42 to the nearest tenth of an inch.

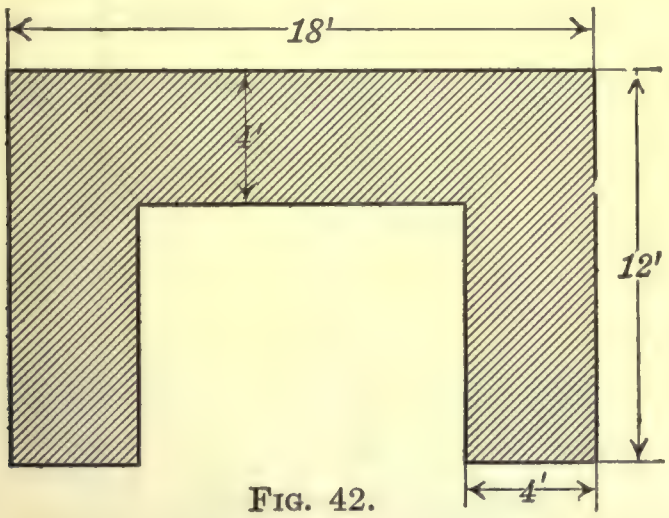


FIG. 42.

(*a*) What is the scale used?

(*b*) Find the total area of the shaded portion.

43. Figure 43 is the ground plan of an apartment house. Find the total area covered.

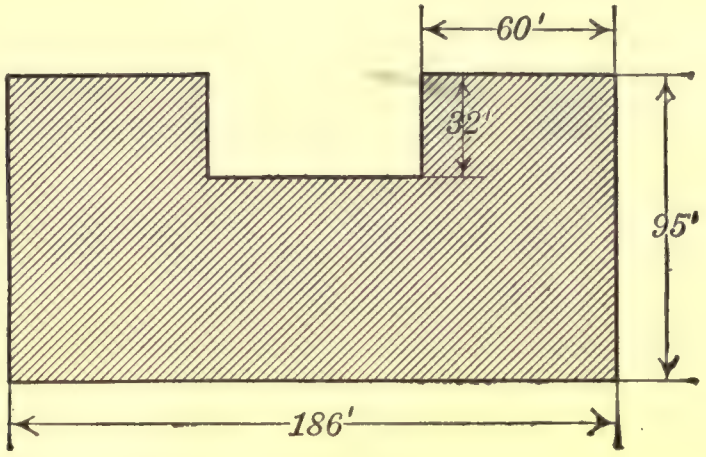


FIG. 43.

44. Figure 44 represents the cross section of an iron beam used in building construction. Find the number of square inches in the area to the nearest tenth of a square inch.

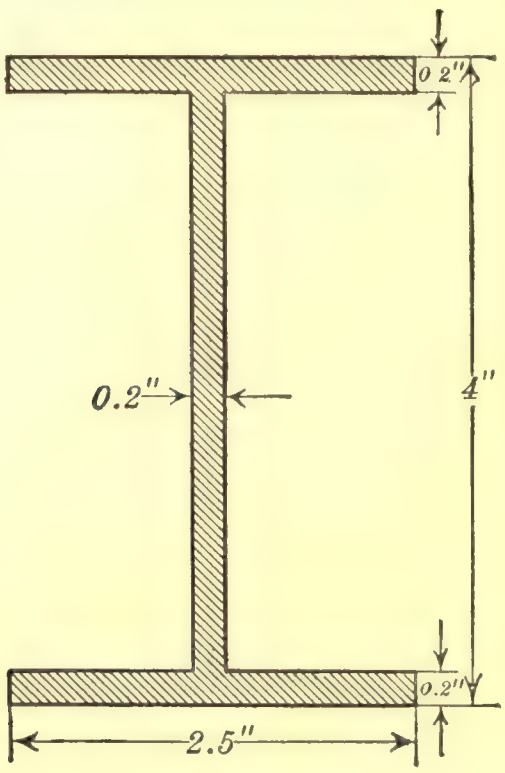


FIG. 44.

III. Squares

EXERCISES

§ 54. 1. Write the formula for the statement:

The area of a square is equal to the square of one side.

The formula is $A = s^2$, where

A = the area of a square (in square units),

s = one side (in linear units).

s^2, read "square of s," means $s \times s$.

2. $s = 15''$, $A = ?$

3. $s = 12\frac{1}{2}''$, $A = ?$

4. $s = 15.2''$, $A = ?$

5. $s = 4.72''$, find A to the nearest tenth of a square inch.

6. $s = 7.34''$, find A to the nearest tenth of a square inch.

7. $A = 25$ sq. in., $s = ?$

Solution.

Placing 25 for s, in the formula, $A = s^2$,

① $25 = s^2$ [Since s^2 means $s \times s$, you find the number that multiplied by itself gives 25.

② $5 = s$ That number, 5, is called the *square root* of 25, hence $5 = s$.] *Ans.* $s = 5''$.

8. $A = 144$ sq. ft., $s = ?$

Solution. Check.

Placing 144 for A, $144 = 12 \times 12$

① $144 = s^2$

② $12 = s$ ① $\sqrt{}$ *Ans.* $s = 12'$.

[The symbol, ① $\sqrt{}$, means that the square root of each member of ① is taken.]

Copy the following table and fill in the required values.

FORMULA: $A = s^2$.

	s	EST.	A
9.	$8\frac{1}{2}''$		
10.	$1'\ 6''$		sq. ft.
11.	$3.4'$		
12.	$5'\ 9''$		
13.			36 sq. in.
14.			64 sq. in.
15.			81 sq. in.
16.			121 sq. in.
17.			400 sq. ft.
18.			900 sq. ft.
19.			625 sq. ft.
20.			2,500 sq. ft.
21.			4,900 sq. ft.
22.			160,000 sq. ft.
23.			12.25 sq. ft.
24.			42.25 sq. ft.

§ 55. Square Root. The numbers that expressed the areas given in Exs. 13 to 24 were perfect squares, hence their sides could be obtained exactly. However, the areas of squares are not likely to be expressed in numbers of such sort that you can get the exact square root. Divide each of the values of A given in the table by the value of s obtained. Note the value of the quotient. The square root of any one of the numbers is seen to be one of the *two equal factors* of the number.

If a number is divided by its square root, the quotient is equal to the divisor. So in order to find the square root of a number, estimate the number to use for your divisor. In order to make close estimates it is necessary to make and use a table of certain squares.

Write the squares of all numbers from 1 through 20; also the squares of 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100. Arrange these squares in a table.

By comparing the square of 2 with the square of 20, the square of 3 with the square of 30, etc., you will see that when you annex ***one*** zero to the number to be squared, you must annex ***two*** zeros to the square of the number.

EXAMPLE 1. A = 89 sq. in., s = ? (to three figures)

SOLUTION.

Placing 89 for A, in the formula, $A = s^2$,

$$89 = s^2$$

To find s, the square root of 89 is required. It lies between 9 and 10, for $9^2 = 81$ and $10^2 = 100$; 89 is a little less than halfway between 81 and 100, hence its square root is about 9.4.

```
      9.46
9.4)89.00
    846
    440
    376
     640
     564
      76
```

The divisor is 9.4, the quotient is 9.46.

Hence the square root of 89 lies between 9.4 and 9.46. Its value then is very nearly 9.43, which is just halfway between the divisor used and the quotient.

Prove that 9.43 is very nearly the square root of 89 by dividing 89 by 9.43.

EXAMPLE 2. A = 3 sq. in., s = ? (to three figures).

SOLUTION.

Placing 3 for A in the formula, $A = s^2$,

① $3 = s^2$

② $\sqrt{3} = s$. [The symbol $\sqrt{\ }$ means *square root*.]

The square root of 3 evidently lies between 1 and 2. To aid you in getting a closer estimate, write the number

3.00, so as to have a *pair* of figures to the right of the decimal point. This lies between 2.89, which is 1.7^2 and 3.24, which is 1.8^2. It is nearer 1.7^2 than 1.8^2, hence let 1.72 be the divisor.

$$
\begin{array}{r}
1.74 \\
1.72\overline{)3.0000} \\
\underline{172} \\
1280 \\
\underline{1204} \\
760 \\
\underline{688} \\
72
\end{array}
$$

Since the quotient is 1.74, the square root is very nearly 1.73, which is halfway between 1.72 and 1.74.

To check, divide 3 by 1.73, and note the quotient.

To get the square root of a number to three figures, you must estimate your square root closely enough so that the quotient and divisor do not differ by more than *one* in the *second* figure. If they do differ by more than one, use a new divisor halfway between them.

EXERCISES

Copy the following table and fill in the required values.

FORMULA: $A = s^2$.

	s	EST.	A		s	EST.	A
1.		3.5″	12.5 sq. in.	11.			2000 sq. in.
2.			30 sq. in.	12.			5 sq. ft.
3.			45 sq. in.	13.			50 sq. ft.
4.		2.7″	7 sq. in.	14.			500 sq. ft.
5.			12 sq. in.	15.			5000 sq. ft.
6.			65 sq. ft.	16.			1 sq. in.
7.			93 sq. ft.	17.			10 sq. in.
8.			2 sq. in.	18.			100 sq. in.
9.			20 sq. in.	19.			1000 sq. in.
10.			200 sq. in.	20.			$4\frac{1}{2}$ sq. ft.

21. $A = s^2$, $s =$ what in terms of A? *Ans.* $s = \sqrt{A}$.

22. Figure 45 is the plan of a hollow square.

(*a*) Find the area of the large square.

(*b*) Find the area of the small square.

(*c*) Find the area of the shaded part.

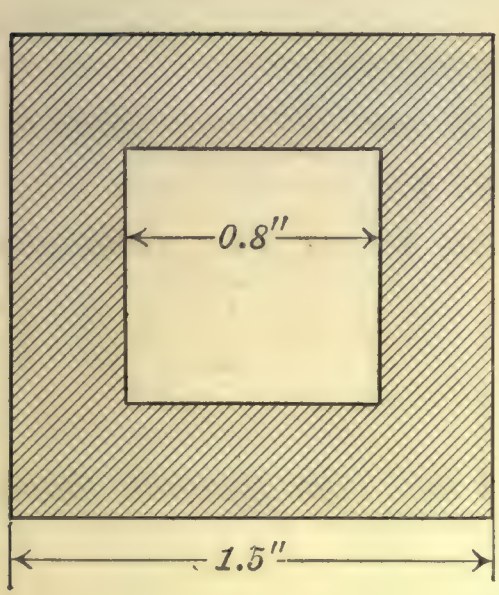


Fig. 45.

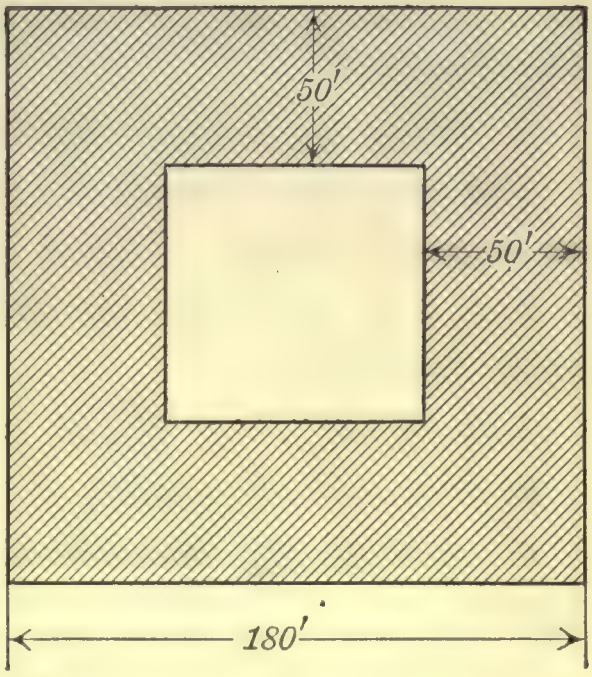


Fig. 46.

23. Figure 46 is the ground plan of a square building with a square courtyard inside.

(*a*) Find the total area covered by the building and the courtyard.

(*b*) Find the area covered by the foundations of the building.

24. A square building covers 17,424 sq. ft. How long is it?

25. A square field contains $3\frac{1}{2}$ acres. How many rods long is it?

IV. Triangles

EXERCISES

§ **56.** **1.** Write the formula for the statement:

The area of a parallelogram is equal to the product of its base and height. (Use the letters A, b and h.)

2. The area of a triangle bears what ratio to the area of a parallelogram that has the same base and height as the triangle?

3. Write the formula for the statement:

The area of a triangle is equal to one half the product of its base and height. (Use A, b and h.)

For Exs. 4–9 use the formula of Ex. 3. Estimate each result first.

4. $b = 12\frac{1}{2}''$, $h = 6''$, $A = ?$

5. $b = 5.4''$, $h = 3.7''$, $A = ?$

6. $A = 45$ sq. in., $h = 7''$, $b = ?$ (to nearest third figure).

Solution. Placing 45 for A and 7 for h, in the formula,

$$A = \tfrac{1}{2}bh,$$

① $\quad 45 = \dfrac{7\,b}{2}$ $\qquad$ (Estimate = 13″)

② $\quad 90 = 7\,b$ $\qquad$ ① × 2

③ $\quad 12.86 = b$ $\qquad$ ② ÷ 7

Check.

$$45 \overset{?}{=} 7 \times \frac{12.86}{2}$$

$45 = 45.01$ $\qquad$ *Ans.* $b = 12.9''$

7. $A = 18$ sq. ft., $b = 3\frac{1}{2}'$, $h = ?$ (to nearest third figure).

8. $A = 6\frac{1}{4}$ sq. in., $h = 2\frac{1}{4}''$, $b = ?$ (to nearest third figure).

9. $A = 48$ sq. in., $b = 11''$, $h = ?$ (to nearest third figure).

10. $A = \frac{bh}{2}$. $b =$ what in terms of A and h? (Multiply each member by 2 first.)

11. $A = \frac{bh}{2}$. $h =$ what in terms of A and b?

12. Measure the dimensions of the triangle in Fig. 47.

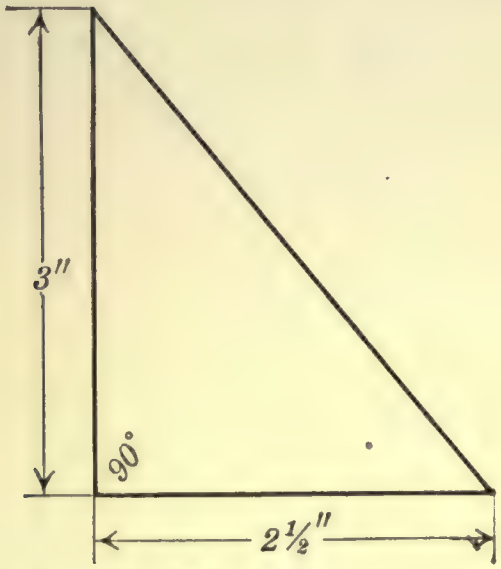


FIG. 47.

(*a*) What is the scale used?

(*b*) Find the area of the triangle.

13. With your protractor and ruler draw a right triangle, having the sides that include the right angle $4''$ and $3''$ long. Measure the hypotenuse. Add the square of 4 to the square of 3, and compare the result with the square of the number of inches in the hypotenuse.

14. With your protractor and ruler draw a right triangle having the sides that include the right angle $1.2''$ and $.9''$ long. Measure the hypotenuse; compare the square of this measure with the sum of the squares of 1.2 and .9.

15. In Fig. 48 the squares are drawn on each of the sides of the right triangle ABC. Show that the area of

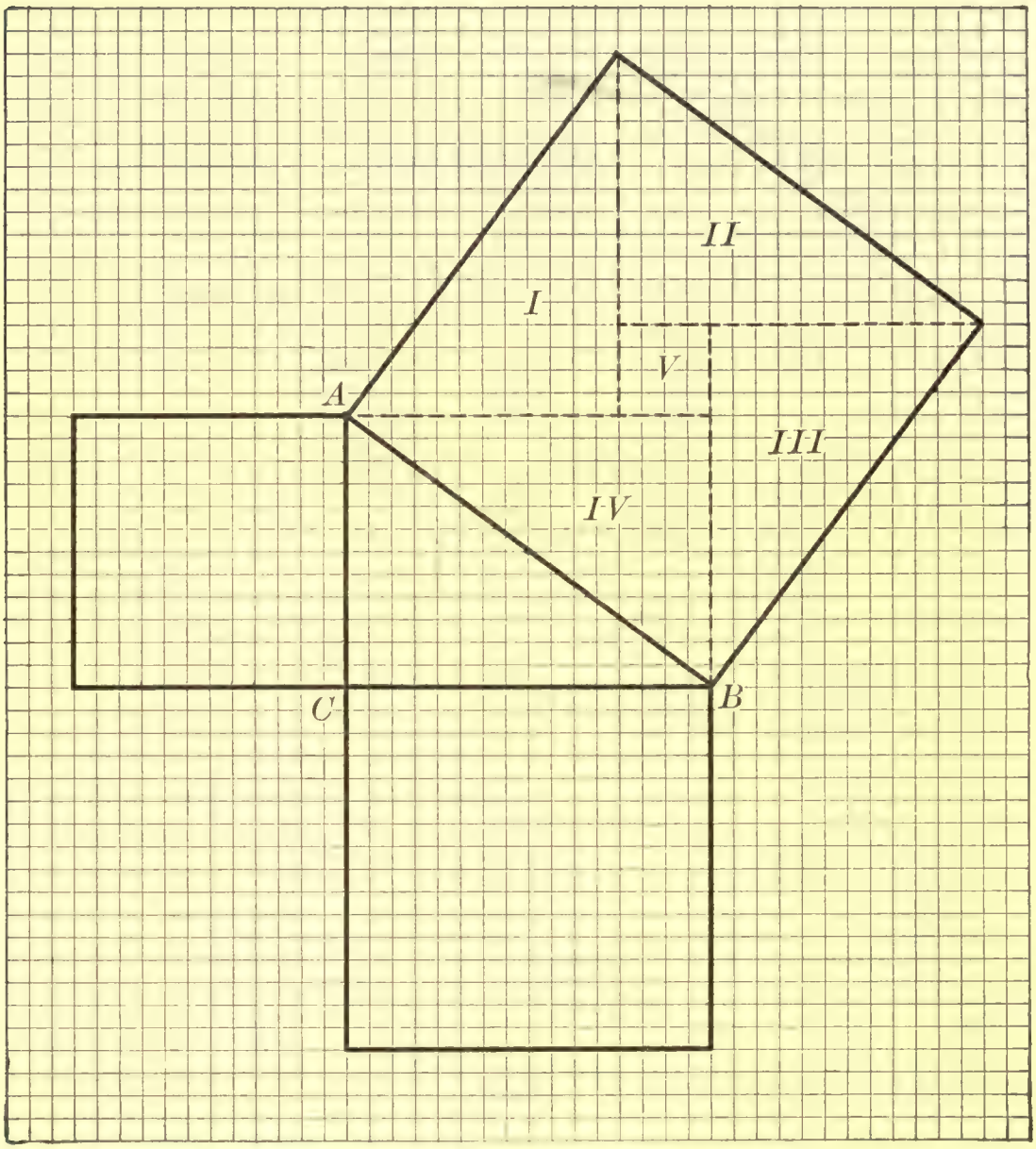


FIG. 48.

the square on the hypotenuse is equal to the sum of the areas of the squares on the two legs.

SOLUTION.

The area of the square on $AC = 12 \times 12 = 144$

The area of the square on $BC = 16 \times 16 = \underline{256}$

400

The area of right $\triangle$ I $= \frac{12 \times 16}{2} = 96$

The area of right $\triangle$ II $= 96$

The area of right $\triangle$ III $= 96$

The area of right $\triangle$ IV $= 96$

The area of square V $= 16$

The area of the square on $AB = 400$

Hence, the area of the square on AB = the area of the square on AC + the area of the square on BC.

16. Write the formula for the statement:

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.

This statement is known as the ***Theorem of Pythagoras.*** The formula is:

$$c^2 = a^2 + b^2,$$

where

c = the hypotenuse of a right $\triangle$ (in linear units),

a = one leg (in linear units),

b = the other leg (in linear units).

17. Write the formula for the statement:

The square of one leg of a right triangle is equal to the square of the hypotenuse minus the square of the other leg. (Use the same letters as in Ex. 16.)

18. The two legs of a right triangle are 5″ and 12″ long; find the hypotenuse.

SOLUTION.

Placing 5 for a and 12 for b, in the formula,

$$c^2 = a^2 + b^2,$$

① $c^2 = 5^2 + 12^2$ (Estimate = 13″)

② $c^2 = 169$ ① ≡

③ $c = 13$ ② $\sqrt{}$

Ans. $c = 13$″.

19. The hypotenuse of a right triangle is 20″ and one leg is 9″; find the other leg to the nearest third figure.

SOLUTION.

Placing 20 for c and 9 for b, in the formula,

$$a^2 = c^2 - b^2,$$

①	$a^2 = 20^2 - 9^2$	(Estimate = 18″)
②	$a^2 = 400 - 81$	① ≡
③	$a^2 = 319$	② ≡
④	$a = 17.9$	③ √

Ans. $a = 17.9''$

Copy the following table and fill in the required values.

FORMULAS: $c^2 = a^2 + b^2$, or $a^2 = c^2 - b^2$, or $b^2 = c^2 - a^2$.

	c	a	b	EST.
20.		8″	6″	
21.		10″	24″	
22.		5″	9″	10″
23.		16″	7″	
24.	15″	12″		
25.	25″		15″	
26.	14″	10″		
27.	16″		11″	
28.		11″	9″	
29.	30″	20″		
30.		8″	12″	
31.	8″		5″	
32.	18″	12″		
33.	35″		10″	
34.		22″	15″	

35. Measure the length and width of the sheet of paper on which you are working (if rectangular). From these measurements compute the length of the longest straight line that you can draw on the paper. Test the accuracy of your result by measuring the distance computed.

36. Figure 49 is a rectangle. Find the length of the diagonal. Check your result by measuring the diagonal

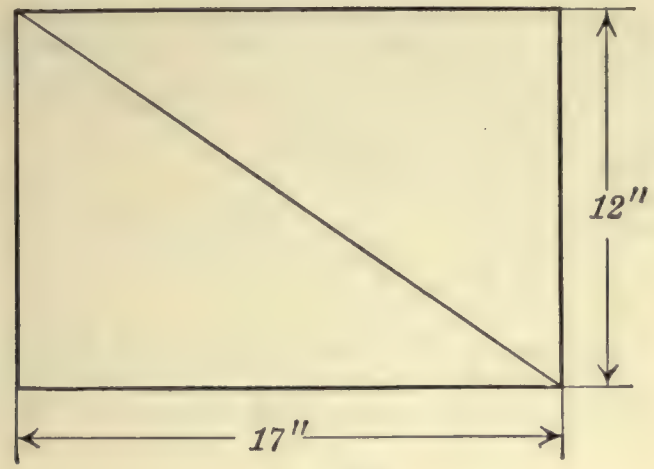


FIG. 49.

to the nearest tenth of an inch. Using the formula $c^2 = a^2 + b^2$, place 17 for a and 12 for b, then solve for c.

37. Find the diagonal of a square, if each side is 8 inches (to the nearest third figure).

38. A baseball diamond is 90 feet square. How far is it from first to third base (to the nearest foot)?

39. A window is 20 feet from the ground; how long must a ladder be to reach to the window, if the foot of the ladder is 8 feet out from the building?

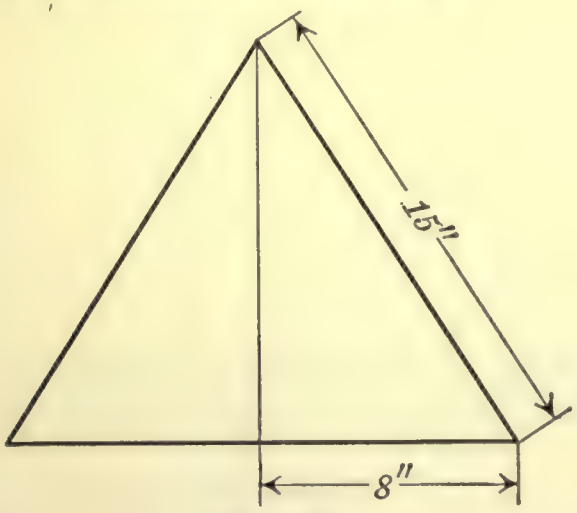


FIG. 50.

40. Find the altitude of the triangle in Fig. 50.

V. Trapezoids

EXERCISES

§ **57.** **1.** Write a formula for the statement:

The area of a trapezoid is equal to one half the sum of its bases multiplied by its height.

The formula is:

$$A = \tfrac{1}{2}(b_1 + b_2)\,h,$$

where

A = the area of a trapezoid (in square units),

b_1 and b_2 = the parallel sides or bases (in linear units),

h = the height (in linear units).

For Exs. 2–7, use the formula of Ex. 1.

Find the results to the nearest third figure, estimating first.

2. $b_1 = 8''$, $b_2 = 9''$, $h = 5''$, $A = ?$

3. $b_1 = 5\frac{1}{2}''$, $b_2 = 7\frac{1}{2}''$, $h = 9\frac{1}{2}''$, $A = ?$

4. $b_1 = 8.4''$, $b_2 = 6.3''$, $h = 7.2''$, $A = ?$

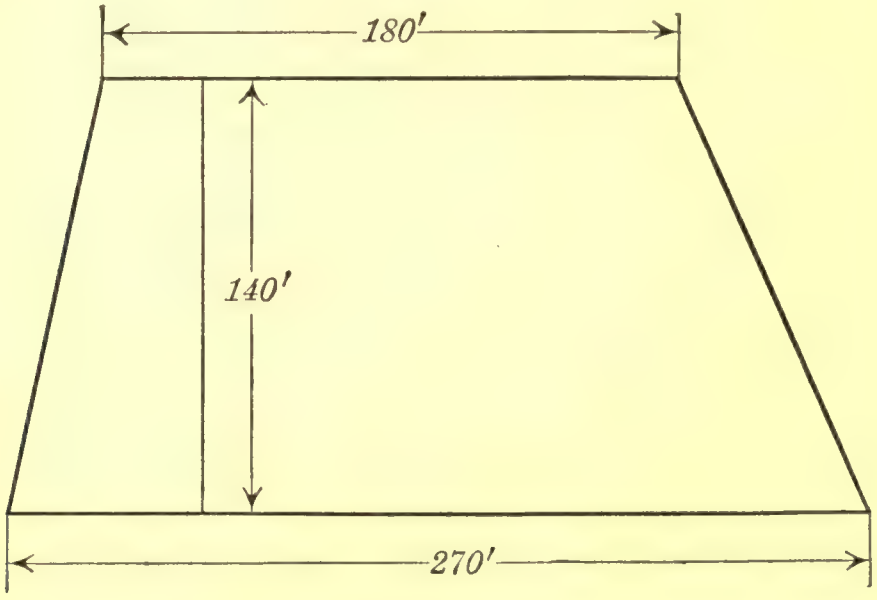


Fig. 51.

5. Figure 51 is the plan of a lot of land in the shape of a trapezoid. Find its area.

6. Figure 52 is the plan of a lot of land in the shape of a trapezoid. (Scale: 1″ for 100′.)

(*a*) Make a copy of the plan, filling in the dimensions.

(*b*) Find the area of the lot.

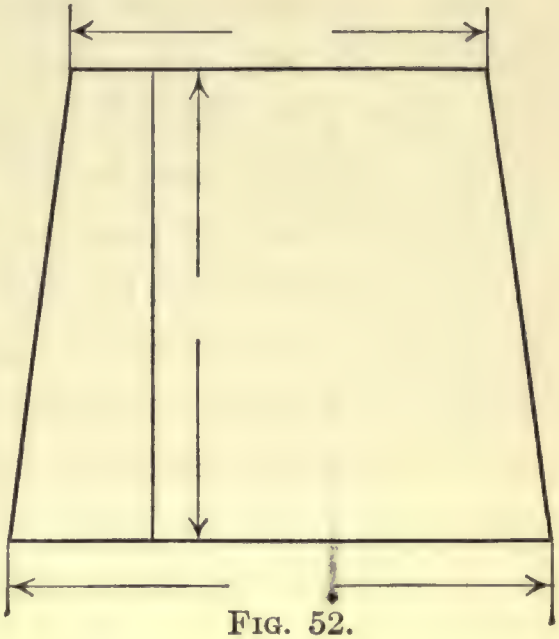


FIG. 52.

7. Figure 53 is the plan of a city block.

(*a*) Find the number of square feet in each lot.

(*b*) At \$5.00 per square foot, what is the cost of lot *A*?

(*c*) At \$4.50 per square foot, what is the cost of lot *C*?

(*d*) Lot *E* costs \$29,450. What is the price per square foot?

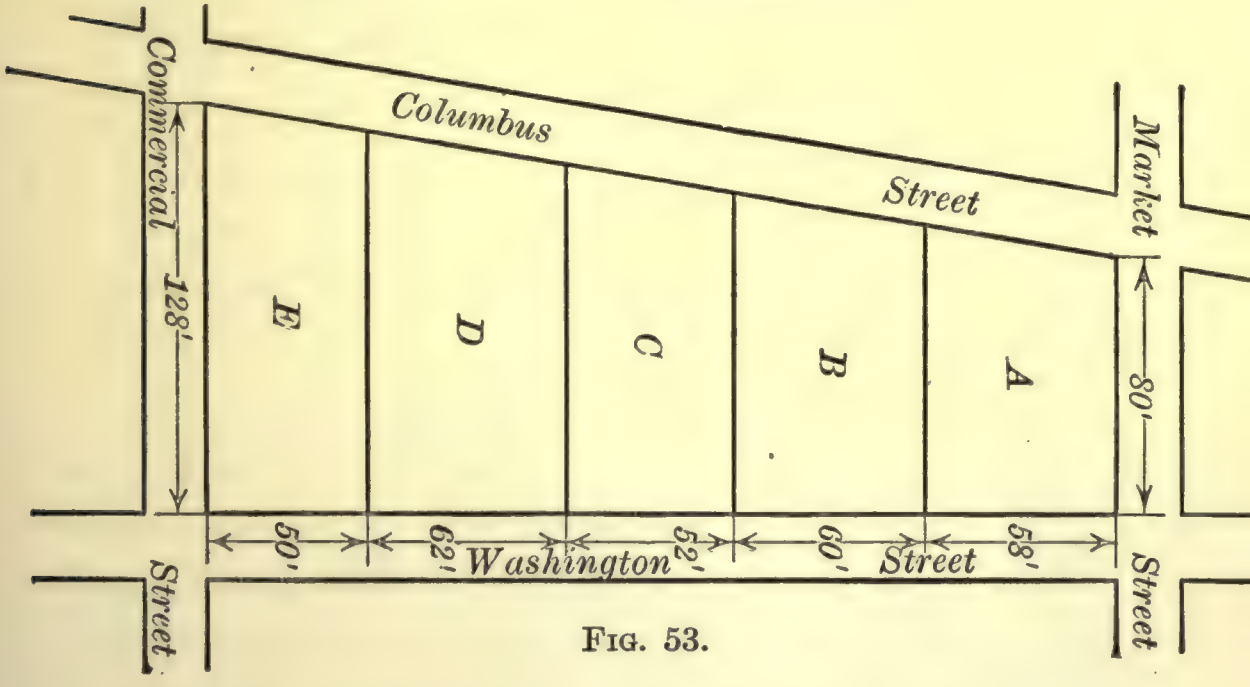


FIG. 53.

VI. Approximate Products

§ 58. In some of the exercises in this chapter, you were required to find the result to three figures only (Exs. 5–8, page 136; Exs. 2–4, page 142; etc.). The reason for this will appear in this section. Some numbers are obtained by counting and some by measuring. When we say that there are 36 desks in a certain room, we mean exactly 36; no more, no less. When we measure the length of a room and say it is 24.6′ long, we generally mean that this number of feet is nearer the length of the room than 24.5′ or 24.7′. The length might be nearly 24.65′ or slightly more than 24.55′. If we wanted to find the area of the floor of a room in square feet, we would probably measure the length and width of the room to the nearest tenth of a foot, and then compute the area from these measurements.

Example 1. The length of a room is 24.6′ and the width is 21.4′. Find the area of the floor in square feet.

Formula. $A = bh$

$A = 24.6 \times 21.4$

Specimen Solutions.

A. The work appears as follows, if the customary method of multiplication is used:

$$\begin{array}{r} 24.6 \\ 21.4 \\ \hline 984 \\ 246 \\ 492 \\ \hline 526.44 \end{array}$$

Estimate

$20 \times 25 = 500$

The area of the floor appears to be 526 sq. ft. to the nearest square foot.

B. If the order of multiplying is reversed; that is, if 24.6 is multiplied by the left-hand digit of the multiplier first, the work will appear as follows:

24.6		
21.4		Estimate
492.	(20 × 24.6)	20 × 25 = 500
24.6	(1 × 24.6)	
9.84	(.4 × 24.6)	
526.44		

In this method the decimal point is located in the first partial product to correspond with the estimate.

NOTE. A more complete explanation of this method of multiplying can be found in Chapter I, First Course.

C. The measurements of the floor, of which we are finding the area, were made only to the nearest tenth of a foot. It will now be shown that this area cannot be accurate beyond the third figure, and that even the third figure is in doubt.

24.6	
21.4	Estimate
492.**	20 × 25 = 500
24.6*	
9.84	
526.44	

The area of the floor is computed as 526.44 sq. ft. In the first partial product, however, the places marked with stars (*) are not necessarily zeros. Had the length been measured to the nearest hundredth of a foot, some figure would have appeared after 492 in the column following the decimal point. The stars show that the figures in these columns are doubtful. In fact the first partial

product, 492, might be either 491 or 493; for 20 times 24.55 is 491, and 20 times 24.64 is 493 to the nearest third figure. (Any measurement between 24.55′ and 24.64′ would be called 24.6′, to the nearest tenth.)

In the final product, therefore, the figures that appear in the doubtful columns (which happen, in this example, to be at the right of the decimal point) are of little use, as their retention does not make the answer any more accurate than it would have been without them. This final product might be as small as 524.1 (21.35 × 24.55), or as large as 528.3 (21.34 × 24.64), depending upon the data. Hence the *third* figure of this answer is in *doubt*, and we waste labor by keeping the figures in the partial products that do not appear in our final product.

D. If we cut the partial products so as to retain only those figures necessary for the final product, the work appears as follows:

$$
\begin{array}{rl}
 & \phantom{\text{Estimate}} \\
24.6' & \\
21.4 & \text{Estimate} \\
\hline
492. & 20 \times 25 = 500 \\
25. & (1 \times 24 + 1. \quad \text{See Note 1.}) \\
10. & (.4 \times 20 + 2. \quad \text{See Note 2.}) \\
\hline
527. &
\end{array}
$$

Note 1. ($1 \times 24.6 = 24.\acute{6}.$) As the 6, if written, would come in a doubtful column (See Solution *C*), we do not write it, but we carry 1 to 24 (since 6 is nearer 10 than 0). We place the mark (′) over the 6 in the multiplicand. This is to show that we have not written a figure in the partial product that would come in a doubtful column, but that we have noted, in multiplying, its effect on the next column to the left.

NOTE 2. ($.4 \times 24.\acute{6} = 9.84$.) As the 8 and the 4, if written, would both come in doubtful columns (See Solution *C*), we do not write them; but, in multiplying 20 by .4, we notice that we had 2 to carry (since $4 \times 4 = 16$, which is nearer 20 than 10). We now place a mark (′) over the 4 in the multiplicand to indicate again that we have not written a figure that would come in a doubtful column; but that we have noted, in multiplying, its effect on the next column to the left.

NOTE 3. Measurements, like the above, made to the nearest third figure, give a result not reliable beyond three figures, hence figures beyond the third may be neglected. Even the third figure may not be reliable. If the first three figures of a result are reliable, such a result is said to be of *three-figure accuracy*.

EXAMPLE 2. A piece of paper is 8.42″ long and 7.36″ wide. How many square inches does it contain? (Find your result to tenths of a square inch.)

FORMULA. $A = bh$

$A = 8.42 \times 7.36$

SPECIMEN SOLUTION.

If the order of multiplying is reversed, the work appears as follows. It is customary to retain all figures in the first partial product.

$8.4\acute{2}$		Estimate
7.36		$7\frac{1}{2} \times 8 = 60$
58.94	(7×8.42)	
2.53	$(.3 \times 8.4\acute{2}.$	See Note 1.)
.50	$(.06 \times 8.4\acute{2}.$	See Note 2.)
62.0		*Ans.* $A = 62.0$ sq. in.

NOTE 1. ($.3 \times 8.4\acute{2}$.) Place mark (′) over 2; multiply 8.4 by .3 and add 1 ($3 \times 2 = 6$, which is nearer 10 than 0).

NOTE 2. ($.06 \times 8.\acute{4}$.) Place mark (′) over 4; multiply 8 by .06 and add 2 ($6 \times 4 = 24$, which is nearer 20 than 30).

NOTE 3. Since the result was required to tenths, it was necessary to write 0 after the decimal point.

EXERCISES

A. By the above method, in each of the following exercises, find the product to three figures.

1. 5.14×24.3
2. 9.27×2.34
3. 20.7×3.42
4. 19.2×24.6
5. 5.24×3.17
6. 92.5×3.14
7. 45.3×18.4
8. 9.81×7.07
9. 3.09×48.7
10. 7.05×6.07
11. 885×3.07
12. 978×4.09
13. 9.90×3.08
14. $879 \times .205$
15. $67.8 \times .907$
16. 7.65×4.28

B. In each of the following exercises, find the product to four figures.

1. 37.38×34.67
2. 27.49×43.58
3. 496.3×13.57
4. 47.38×71.17
5. 3.489×1.192
6. 7.854×34.24
7. 25.75×3.142
8. 12.37×14.59
9. 301.7×180.6
10. 27.09×30.09

VII. Circles

§ 59. A ***Circle*** is a curved line all points of which are equidistant from a point within called the center.

EXERCISES

1. Write the formula for the statement:

The circumference of a circle is equal to 2π times its radius.

The letter π is used in all civilized countries to denote the number

$$\pi = 3.14159265 \cdots$$

Roughly, $\pi = 3\frac{1}{7}$, or 3.14 to three figures.

2. Write the formula for the statement:

The area of a circle is equal to π times the square of its radius.

3. Write a formula for the diameter of a circle in terms of its radius.

4. Write a formula for circumference of a circle in terms of its diameter.

5. $A = 500$ sq. in., $r = ?$ $c = ?$

Solution.

Placing 500 for A and 3.14 for π, in the formula,

$$A = \pi r^2,$$

① $500 = 3.14\, r^2$ (*Est.* $r = 13''$)

② $159.2 = r^2$ ① ÷ 3.14

③ $12.6 = r$ ② √

Ans. $r = 12.6''$.

$c = 2\pi r$

① $c = 2 \times 3.14 \times 12.6$ (*Est.* $c = 80''$)

② $c = 79.1$ ① ≡

Ans. $c = 79.1''$.

Copy the following table and fill in the required values, getting each to the nearest third figure.

FORMULAS: $d = 2r$, $c = 2\pi r$, $A = \pi r^2$.

	r	d	c	A	EQUATIONS	ESTIMATES
6.	8″	16″	?	?	$\begin{cases} c = 2 \times 3.14 \times 8 \\ A = 3.14 \times 8^2 \end{cases}$	$c = 50''$ $A = 200$ sq. in.
7.		12.4″				
8.			120″			
9.				225 sq. in.	$\begin{cases} 225 = 3.14\, r^2 \\ c = 2 \times 3.14 \times ? \end{cases}$	$r = 8\frac{1}{2}''$ $c = 50''$
10.	83′					
11.		18.8′				
12.			430′			
13.				127 sq. ft.		
14.			12′ 9″			
15.				132.5 sq. in.		
16.				1000 sq. ft.		

17. Find the cross-sectional area of the inside of a water pipe, if the inside diameter is 15″. (See Fig. 54.)

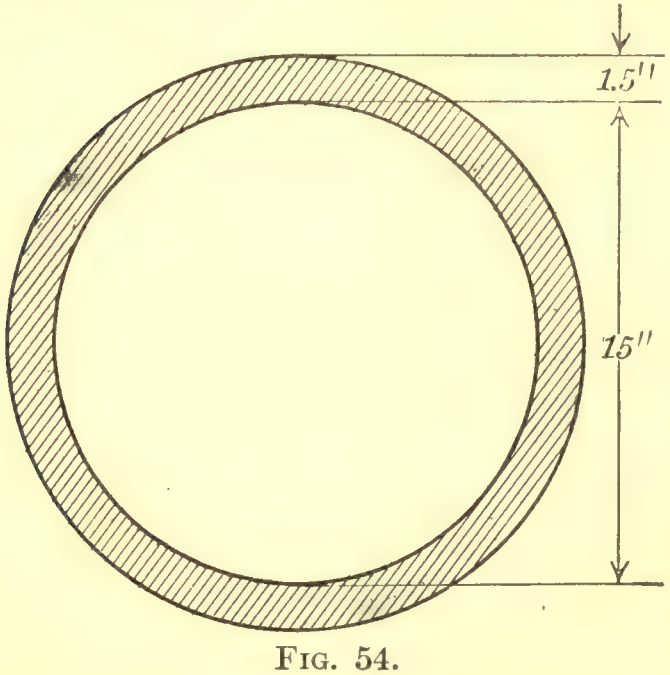


FIG. 54.

18. The iron in this pipe is $1\frac{1}{2}''$ thick.

(*a*) What is its outside diameter?

(*b*) What is the outside cross-sectional area?

(*c*) What is the cross-sectional area of the metal in the pipe (shaded portion)?

19. The diameter of a circular grass plot is 62.4′.

(*a*) Find the distance around it.

(*b*) Find the number of square feet of turf in it.

(*c*) A walk 4′ wide surrounds it. Find the number of square feet of land covered by the grass and walk.

(*d*) Find the area of the walk.

(*e*) The area of the walk is what per cent of the total area (nearest per cent)? What per cent of the area of the turfed part?

20. The side of the square in Fig. 55 is 8″ (scale : 10 to 1)

(*a*) Find the diameter of the circle.

(See page 139.)

(*b*) Find the area of the circle.

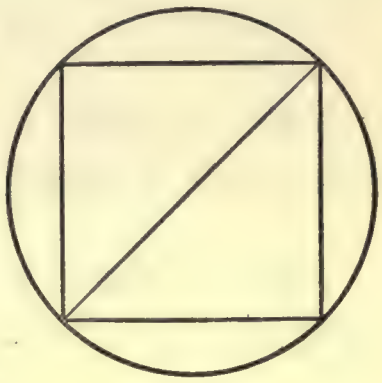


FIG. 55.

21. Find the inside diameter of a circular pipe, if its inside cross-sectional area is 12.3 sq. in.

22. A square grass plot and a circular grass plot both contain 400 sq. yd.

(*a*) Find the length of the fence around the square plot.

(*b*) Find the length of the fence around the circular plot.

(*c*) If the fences are of the same kind, what per cent more would the first cost than the second?

23. The diameter of a circular track is 168′. Find how many laps a runner must make to go a mile.

24. A bicycle wheel, 28″ in diameter, makes how many revolutions in going one mile?

25. Find how many square inches there are in the largest circular piece of paper that can be cut from a square piece having an area of 4 square inches.

26. In the machine shop, rods with square ends are cut from rods with circular ends. To get a square rod each side of which is 2″, what should be the diameter (to the nearest third figure) of the round stock to be used? (This process is called "milling.")

CHAPTER VII

MENSURATION OF SOLIDS

I. Blocks

§ 60. A ***solid*** occupies space. A piece of coal, a book, a baseball, a block, are solids.

§ 61. A ***block*** is a solid having six rectangular faces or surfaces. A block is sometimes called a *rectangular solid.*

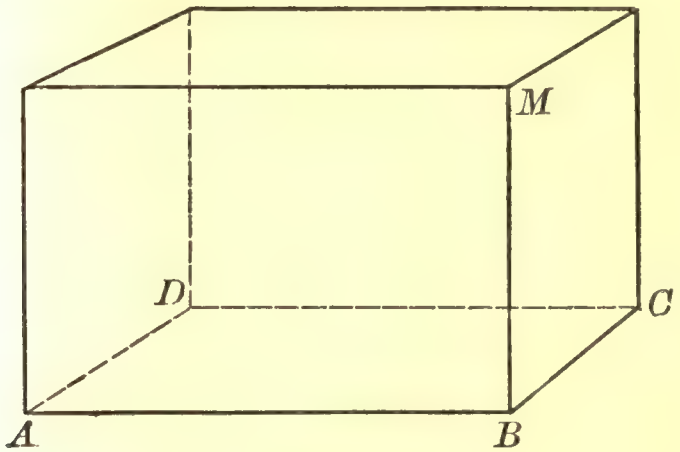


Fig. 56.

In the block in Fig. 56, rectangle $ABCD$ is the *base* and the edge BM is its *height* (or *altitude*).

A ***cube*** is a block having all its faces squares.

§ 62. Entire Surface of a Block. The *entire surface* of a block consists of two ends that are equal rectangles; two sides that are equal rectangles; and the top and bottom that are equal rectangles.

EXERCISES

1. Find the area of the entire surface of a block 8″ long, 6″ wide, and 5″ high.

2. Find the area of the entire surface of a block 12″ long, 8″ wide, and 9″ high.

3. Find the area of the entire surface of a block $5\frac{1}{2}$″ long, $3\frac{1}{2}$″ wide, and $4\frac{1}{2}$″ high.

4. A room is 14′ 6″ long, 12′ wide, and 8′ 6″ high.

(*a*) Find the area of the floor.

(*b*) Find the area of the ceiling.

(*c*) Find the total area of the four walls.

§ 63. Entire Surface of a Cube. —

EXERCISES

1. Find the area of the entire surface of a cube if its edge is 12″.

2. Find the area of the entire surface of a cube if its edge is 3.4″.

3. Find the area of the entire surface of a cube if its edge is 1′ 6″.

4. Write the formula for finding the area of the entire surface of a cube.

NOTE. Let T = the area of the entire surface, or the total area of the cube, and let e = one edge.

§ 64. Volume of a Block. A *cubic inch* is the contents of a cube 1″ long, 1″ wide, and 1″ high. *A cubic foot* is the contents of a cube 1′ long, 1′ wide, and 1′ high. Since 1 foot (length) = 12 inches, and 1 square foot (area) = 12 × 12 square inches, it follows that 1 cubic foot = 12 × 12 × 12 cubic inches.

TABLE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).

The drawing in Fig. 57 represents a block 4″ long, 3″ wide, and 3″ high. This block consists of 3 layers, each

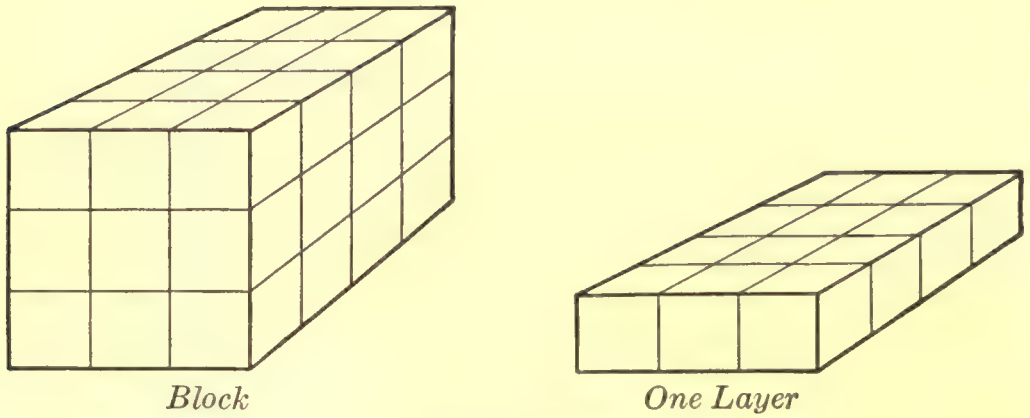


FIG. 57.

made up of 12 little blocks (4 × 3 blocks). Each little block represents one cubic inch. There are in the block 3 × 12 cubic inches or 36 cubic inches.

The number of cubic inches in the block is the ***volume*** of the block. If V stands for the volume of the block (in cubic units), l for the length (in linear units), w for the width (in linear units), and h for the height (in linear units), then

V (the number of cubic units) = lwh cubic units.

NOTE. It will be assumed that this formula, and others that follow, can be used when the dimensions are fractional.

EXERCISES

1. Find the contents in cubic inches of a block 14″ long, 12″ wide, and 8″ high.

2. Find the contents in cubic feet of a rectangular tank 12′ long, 10′ wide, and 8′ deep.

3. How many cubic yards must be excavated to make a cellar 35′ by 40′ and 6′ deep?

4. The floor of a coal bin is 15′ long and 8′ wide. The bin is 7′ in height.

(*a*) Find the contents of the bin in cubic feet.

(*b*) How many tons of coal will this bin hold? (Allow 35 cu. ft. for 1 ton.)

5. The inside dimensions of a rectangular tank, full of water, are 12′, 8′, and 6′.

(*a*) How many cubic feet of water are in the tank?

(*b*) One cubic foot of water weighs 62.5 lb. What is the weight of the water in the tank?

6. How many cubic feet of earth are removed in digging a ditch 1.5′ wide, 2′ deep, and 200′ long? How many cubic yards?

7. A rectangular watering trough is made 10 ft. long, 30 in. wide, and 18 in. deep.

(*a*) How many cubic feet of water will it hold?

(*b*) How many gallons will it hold, allowing $7\frac{1}{2}$ gallons to the cubic foot?

NOTE. All the units should be of the same kind before using the formula.

8. A rectangular water tank, open at the top, is to be lined with copper. It is 24″ long, 18″ wide, and 12″ deep.

(*a*) How many square inches of copper are needed, if there is no allowance for waste or overlapping?

(*b*) How many square inches are needed, if 5% is added for waste and overlapping?

(*c*) How many gallons of water will the tank contain (to the nearest tenth of a gallon), if 231 cubic inches are allowed for a gallon?

§ 65. Volume of a Cube. The length, width, and height of a cube are all equal, so the volume is found by multiplying together three equal numbers. This is called *cubing* a number.

If V stands for the volume of a cube (in cubic units) and e stands for the length of one edge (in linear units), then

V (the number of cubic units) $= e \times e \times e$ cubic units, or

$\boldsymbol{V = e^3}$ (read " the cube of e ").

EXAMPLE. Find the contents in cubic inches of a cube if its edge is 6″.

SOLUTION.

$V = e^3$

① $V = 6^3$, or $6 \times 6 \times 6$

② $V = 216$ ① ≡

Ans. $V = 216$ cu. in.

EXERCISES

1. Find the contents in cubic feet of a cubical box whose inside length is 3 feet.

2. Find the contents in cubic inches of a cubical piece of steel $2\frac{1}{2}''$ long.

3. A room, cubical in shape, is 8′ 6″ wide. How many cubic feet of air does it contain?

4. How many cubic inches will 12 cubes occupy, if each one is $2\frac{1}{2}''$ long?

5. Compare the volumes of two cubes whose edges are 2 ft. and 4 ft., respectively.

6. Compare the volume of any cube with the volume of a cube whose edge is twice as long; with one whose edge is three times as long.

§ 66. Density. The ***density*** of a substance is the weight of a cubic unit. It may be expressed in pounds per cubic foot, or in pounds per cubic inch, or in ounces per cubic inch, etc.

EXERCISES

1. Find the weight of a block of ice 1′ by 1′ 6″ by 1′ 3″. The density of ice is 57 lb. per cubic foot. This is the scientific way of saying that a cubic foot of ice weighs 57 lb.

2. The density of steel is .28 lb. per cubic inch. Find the weight of a block of steel 12″ long, 3″ wide, and 2″ thick (to the nearest pound).

3. The density of clay is 75 lb. per cubic foot. What is the weight of clay in a cart even full, if the cart is 8′ long, 4′ wide, and 2′ 6″ deep?

4. In a pile there are 1200 bricks 8″ by 4″ by 2″.

(*a*) How many cubic feet of space will they occupy?

(*b*) If the density of brick is 125 lb. per cubic foot, how much will they weigh (to the nearest pound)?

5. Find the weight of water in a cubical container 2 ft. long (inside measurement). The density of water is 62.5 lb. per cubic foot.

6. Find the weight of a steel cube 8″ long. The density of steel is .28 lb. per cubic inch.

7. A certain marble cube is 15″ long. The density of marble is 168 lb. per cubic foot.

(*a*) What is the weight of the marble cube?

(*b*) What is the density of marble per cubic inch (to the nearest hundredth of a pound)?

8. A cubical tank has its inside dimensions each 18″ long.

(*a*) How many gallons of water will it contain?

(*b*) How much will the water in the tank weigh, when the tank is full?

§ 67. Board Measure. Lumber is measured in *board feet.* A board 1 foot long, 1 foot wide, and 1 inch thick contains 1 *board foot.* A board 2 feet long, 6 inches wide, and 1 inch thick also contains 1 board foot. To find the number of board feet in a piece of lumber:

Multiply the thickness, expressed in inches, by one twelfth of the width, expressed in inches, by the length, expressed in feet.

The formula for measuring rough lumber is:

$$B = t \times \frac{w}{12} \times l,$$

where

B = the number of board feet,
t = the thickness in inches,
w = the width in inches,
l = the length in feet.

NOTE. It is customary in business to assume that $t = 1$ for all lumber less than 1 inch thick.

EXAMPLE 1. Find the number of board feet in a board $2'' \times 8''$, 12′ long.

SOLUTION. $B = t \times \frac{w}{12} \times l$

① $B = 2 \times \frac{8}{12} \times 12$ (*Est.* $B = 15$ bd. ft.)

② $B = 16$ *Ans.* $B = 16$ bd. ft.

EXERCISES

Copy the following table and fill in the required values.

$$\text{Formula: } B = t \times \frac{w}{12} \times l.$$

	t	w	l	Est.	B
1.	$1''$	$12''$	$12'$		
2.	$2''$	$6''$	$10'$		
3.	$3''$	$4''$	$12'$		
4.	$1\frac{1}{2}''$	$6''$	$10'$		
5.	$1''$	$9''$	$16'$		
6.	$1\frac{1}{4}''$	$12''$	$16'$		
7.	$1\frac{1}{4}''$	$10''$	$12'$		
8.	$2\frac{1}{2}''$	$4''$	$12'$		
9.	$\frac{1}{2}''$*	$8''$	$12'$		
10.	$\frac{7}{8}''$	$9''$	$16'$		
11.	$\frac{3}{8}''$	$3''$	$12'$		
12.	$\frac{1}{2}''$	$1\frac{1}{2}''$	$12'$		
13.	$\frac{7}{8}''$	$12''$	$10'$		
14.	$1''$	$8''$	$8'$		
15.	$\frac{1}{2}''$	$8''$	$18''$		
16.	$1\frac{1}{2}''$	$6''$	$16''$		

* See note following formula, page 158.

17. How many board feet are there in 24 pieces, each of which is $1\frac{1}{2}'' \times 4'' \times 14'$?

18. How many board feet are there in 12 pieces, each of which is $2'' \times 4'' \times 16'$?

19. How many board feet are there in 100 pieces, each of which is $\frac{1}{2}'' \times 3'' \times 14'$?

20. How many board feet are there in 120 pieces, each of which is $1\frac{1}{4}'' \times 8'' \times 24''$?

21. A pile of $1''$ boards is $12'$ long, $8'$ wide, and $3'$ high. How many board feet are there in the pile?

22. What is the cost of 100 boards, $1'' \times 9'' \times 16'$ at \$80 per M?

(\$80 per M means \$80 per thousand board feet.)

SOLUTION.

$$\text{Cost (in dollars)} = \cancel{100} \times 1 \times \frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}} \times 16 \times \frac{\overset{2}{\cancel{8}}\ \cancel{80}}{\underset{10}{\cancel{1000}}}$$

$$\text{Cost} = 96$$

Ans. Cost = \$96.

23. Find the cost of 18 timbers, $3'' \times 9'' \times 12'$ at \$40 per M.

24. Find the cost of 50 planks, $1\frac{1}{2}'' \times 10'' \times 16'$ at \$75 per M.

25. Find the cost of 80 boards, $\frac{7}{8}'' \times 3'' \times 12'$ at \$120 per M.

26. Mr. Charles Jones orders from the Cambridge Lumber Co. the following:

12 pieces, $2'' \times 4''$, $14'$ long at \$35 per M.
40 pieces, $\frac{1}{2}'' \times 6''$, $12'$ long at \$80 per M.
60 pieces, $\frac{7}{8}'' \times 9''$, $16'$ long at \$90 per M.

Find the total amount of the bill.

27. You order from the Franklin Lumber Company the following:

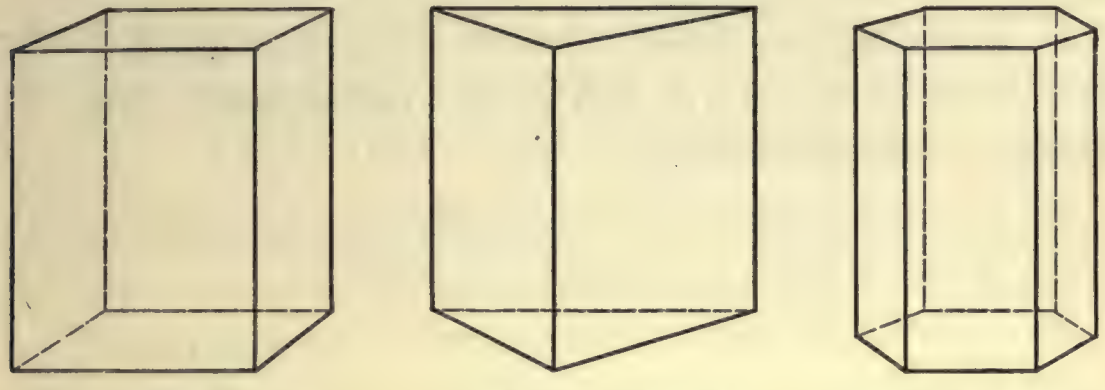
120 pieces, $1\frac{1}{4}'' \times 8'' \times 12'$ at \$80 per M.
150 pieces, $\frac{1}{2}'' \times 8'' \times 16'$ at \$100 per M.
90 pieces, $2'' \times 4'' \times 14'$ at \$60 per M.

(*a*) Find the total amount of the bill.

(*b*) Allow a discount of 5% for cash. Find the amount if cash payment is made.

II. Prisms — Cylinders

§ 68. Right Prisms. A ***right prism*** is a solid having two equal polygons for bases and rectangles for its other faces. A ***block*** is a right prism whose two bases are rectangles. (See Fig. 58.)



Rectangular Prism (Block) *Triangular Prism* *Hexagonal Prism*

Fig. 58.

§ 69. Volume of a Right Prism. The volume of a right prism is equal to the product of the area of its base by its height. This rule is expressed by the formula:

$$V = Bh,$$

where

V = the volume of a right prism (in cubic units),
B = the area of the base (in square units),
h = the height (in linear units).

Note. No attempt will be made to develop the remaining formulas in this chapter.

EXERCISES

(Only *right prisms* will be considered in these exercises.)

1. Find the volume of a prism if its height is 3.2″ and the area of its base is 43.2 sq. in.

2. Find the volume of a prism if its height is $4\frac{1}{2}''$ and the area of its base is $13\frac{1}{2}$ sq. in.

3. The density of marble is 168 lb. per cubic foot. A piece for a column is cut into the shape of a hexagonal prism, and the area of one end is $1\frac{1}{4}$ sq. ft. If the piece is $4\frac{1}{2}'$ long, how much does it weigh?

4. A plate glass window is $8'$ by $12'$ and $\frac{1}{4}''$ thick. The density of plate glass is 172 lb. per cubic foot. Find the weight of the window.

5. A rectangular water tank is $20'' \times 16'' \times 8''$.

(*a*) Find its volume in cubic inches.

(*b*) How many gallons does it contain when full?

(*c*) When full, how much will the water weigh?

§ 70. Cylinders. If you revolve a rectangle about one of its sides, making a complete revolution, the solid formed

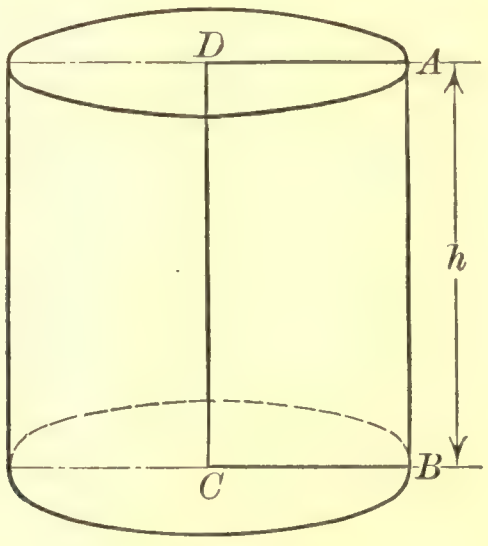


FIG. 59.

is a *cylinder*. The base of this cylinder is a *circle*. In Fig. 59 the cylinder is formed by revolving the rectangle $ABCD$ about its side CD. A cylinder resembles a regular prism which has a very great number of sides. Compare the rules for finding their volumes. (§§ 69 and 72.)

§ 71. Surface of a Cylinder. The curved surface of a cylinder is equal to the product of the circumference of its base by its height. (See Ex. 1, page 149.)

This rule is expressed by the formula:

$$S = 2\pi rh,$$

where

S = the curved surface of a cylinder (in square units),
r = the radius of the base (in linear units),
h = the height (in linear units).
$\pi = 3.14159 \cdots = 3.14$, approximately.

§ 72. Volume of a Cylinder. The volume of a cylinder is equal to the product of the area of its base by its height. (See Ex. 2, page 149.)

This rule is expressed by the formula:

$$V = \pi r^2 h,$$

where

V = the volume of a cylinder (in cubic units),
r = the radius of the base (in linear units),
h = the height (in linear units).
$\pi = 3.14159 \cdots = 3.14$, approximately.

EXAMPLE. Find the area of the curved surface and the volume of a cylinder, if the radius of its base is $5\frac{1}{2}''$ and its height is $6''$ (to the nearest tenth).

SOLUTION.

$S = 2\pi rh$

① $S = 2 \times 3.14 \times 5.5 \times 6$ (*Est.* $S = 200$ sq. in.)

② $S = 207.2$ ① ≡

$V = \pi r^2 h$

① $V = 3.14 \times 5.5^2 \times 6$ (*Est.* $V = 600$ cu. in.)

② $V = 568.9$ ① ≡

Ans. $S = 207.2$ sq. in.,
$V = 568.9$ cu. in.

EXERCISES

Copy the following table and fill in the required values (to the nearest tenth).

FORMULAS: $S = 2\pi rh$, $V = \pi r^2 h$.

	r	h	EST. S	S	EST. V	V
1.	4″	3″	75 sq. in.		150	
2.	8″	15″			cu. in.	
3.	5″	4″				
4.	8″	$3\frac{1}{2}$″				
5.	$6\frac{1}{2}$′	8′				
6.	10.2″	5.3″				
7.	8.7″	12.2″				
8.	$4\frac{1}{2}$′	$5\frac{1}{2}$′				
9.	4′ 6″	8′ 3″				

10. A cylindrical pail is 6″ in diameter (inside) and 15″ high.

(*a*) How many cubic inches are there in its contents?

NOTE. The diameter is given, not the radius.

(*b*) How many gallons will it hold?

11. A cylindrical hot water tank is 4′ high and 1′ 4″ in diameter.

(*a*) How many cubic feet of water will it hold?

(*b*) How many gallons will it hold?

12. A cylindrical cast iron bar is 2″ in diameter and 12′ long. The density of cast iron is 450 lb. per cubic foot.

(*a*) Find the volume of the rod in cubic feet.

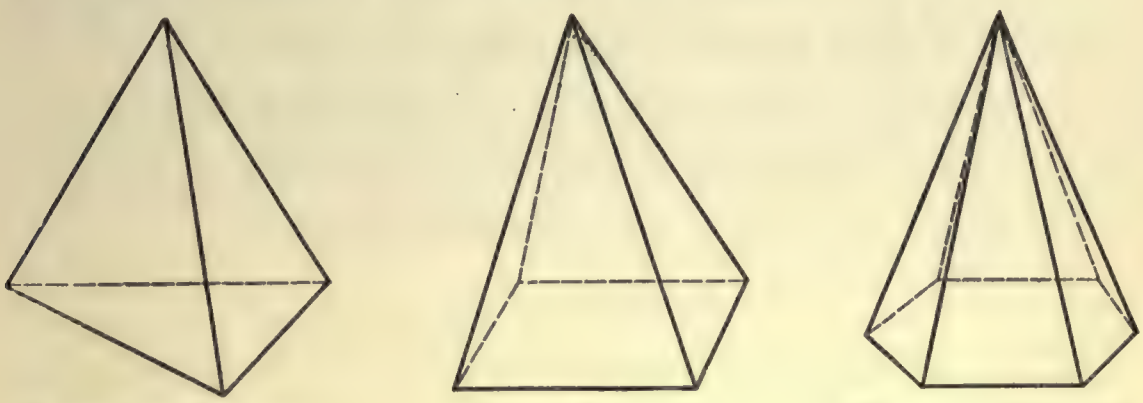
(*b*) Find the weight of the rod.

(*c*) A piece weighing 20 lb. is cut off. What fractional part of the rod is left? What per cent of it?

III. Pyramids — Cones

§ 73. Pyramids. A ***pyramid*** is a solid whose base is a polygon and whose faces are triangles meeting at a common point, called the vertex.

A ***regular pyramid*** has a regular polygon for its base and equal isosceles triangles for its faces.



Triangular Pyramid *Square Pyramid* *Hexagonal Pyramid*

Fig. 60.

The *height* (or *altitude*) of a pyramid is the length of the perpendicular drawn from the vertex to the base.

EXERCISES

[Only *regular pyramids* will be considered in these exercises.]

1. One side of the base of a square pyramid is 4″ long. The height of one of the triangular faces is 6″.

(*a*) Find the area of the base.

(*b*) Find the area of one of the triangular faces.

(*c*) Find the lateral area of the pyramid (sum of areas of triangular faces).

(*d*) Find the entire area of the pyramid.

Note. The entire area is the sum of the areas of the four triangular faces and the base.

2. One side of the base of a square pyramid is $4\frac{1}{2}''$, and the height of one of the triangular faces is $3.2''$.

(*a*) Find the area of the base.

(*b*) Find the area of one of the triangular faces.

(*c*) Find the lateral area.

(*d*) Find the entire area.

NOTE. The height of one of the triangular faces is called the *slant height* of the regular pyramid.

3. One side of the base of a regular hexagonal pyramid is $14.2''$, and the slant height of the pyramid is $12''$.

(*a*) Find the area of one triangular face.

(*b*) Find the lateral area.

§ 74. Volume of a Regular Pyramid. The volume of a regular pyramid is equal to one third of the product of the area of its base by its height.

This rule is expressed by the formula:

$$V = \tfrac{1}{3}\,Bh,$$

where

V = the volume of a regular pyramid (in cubic units),

B = the area of the base (in square units),

h = the height (in linear units).

EXERCISES

1. The base of a pyramid contains 15 sq. ft. and the height is $8'$. Find its volume.

2. One side of the base of a square pyramid is $5'$ and the height is $6'$. Find its volume.

3. The volume of a regular pyramid is 45 cu. ft. The area of its base is 15 sq. ft. Find its height.

4. The volume of a regular pyramid is 250 cu. in. Its height is $45''$. Find the area of its base.

§ 75. Cones. If you revolve a right triangle about one of its legs, making a complete revolution, the solid formed is a ***cone***. The base of this cone is a *circle*.

In Fig. 61, the cone is formed by revolving the right triangle AOC about its leg OA.

AC is the ***slant height*** of the cone.

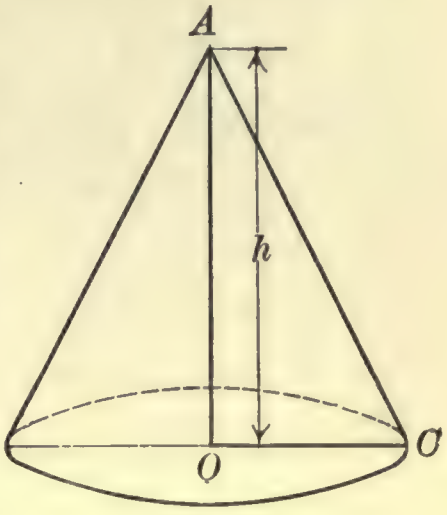


Fig. 61.

§ 76. Surface of a Cone. The curved surface of a cone is equal to one half of the product of the circumference of its base by its slant height.

This rule is expressed by the formula:

$$S = \pi rl,$$

where

S = the curved surface of a cone (in square units),
r = the radius of the base (in linear units),
l = the slant height (in linear units).

§ 77. Volume of a Cone. The volume of a cone is equal to one third of the product of the area of its base by its height.

This rule is expressed by the formula:

$$V = \tfrac{1}{3}\pi r^2 h,$$

where

V = the volume of a cone (in cubic units),
r = the radius of the base (in linear units),
h = the height (in linear units).

A cone resembles a regular pyramid which has a very great number of sides. Compare the rules for finding their volumes. (§§ 74 and 77.)

EXAMPLE 1. Find the area of the curved surface of a cone, if its radius is 7.2″ and its slant height is 9″.

SOLUTION.

$S = \pi\, r\, l$

① $S = 3.14 \times 7.2 \times 9$ (*Est.* $S = 200$ sq. in.)

② $S = 203.5$ ① ≡ *Ans.* $S = 203.5$ sq. in.

EXAMPLE 2. Find the volume of a cone, if its radius is 8.2″ and its height is 6″ (to the nearest tenth).

SOLUTION.

$V = \frac{1}{3}\pi\, r^2\, h$

① $V = \frac{1}{3} \times 3.14 \times 8.2^2 \times 6$ (*Est.* $V = 425$ cu. in.)

② $V = 422.3$ ① ≡ *Ans.* $V = 422.3$ cu. in.

EXERCISES

Copy the following table and fill in the special values required (to the nearest tenth).

FORMULAS: $S = \pi\, r\, l$, $V = \frac{1}{3}\pi\, r^2\, h$.

	r	h	l	EST. S	S	EST. V	V
1.	5″	—	8″	?	?	—	—
2.	6″	4″	—	—	—	?	?
3.	10″	—	$12\frac{1}{2}$′	?	?	—	—
4.	$8\frac{1}{2}$′	12′	—	—	—	?	?
5.	4.4″	—	6.3″	?	?	—	—
6.	2′ 6″	—	4′ 6″	?	?	—	—
7.	3.6″	5.3″	—	—	—	?	?
8.	3′ 6″	4′ 6″	—	—	—	?	?

9. A pile of grain in the form of a cone is 6 ft. in diameter and 3 ft. high.

(*a*) How many cubic feet are there in the pile?

(*b*) Allowing $1\frac{1}{4}$ bushels to the cubic foot, how many bushels are there in the pile?

10. The slant height of a conical tent is to be 10 feet. The diameter of its base is to be 12 feet.

(*a*) How many square yards of canvas are needed for it, making no allowance for the seams and for waste?

(*b*) How many square yards are needed if 10% is added for the seams and for waste?

11. An isosceles triangle is revolved about its altitude so as to form a cone. The altitude of the triangle is 8″, and its base is 12″ (Fig. 62).

(*a*) Find one of the equal sides of the triangle. (See page 139.)

(*b*) Find the area of the curved surface of the cone formed.

(*c*) Find the volume of the cone.

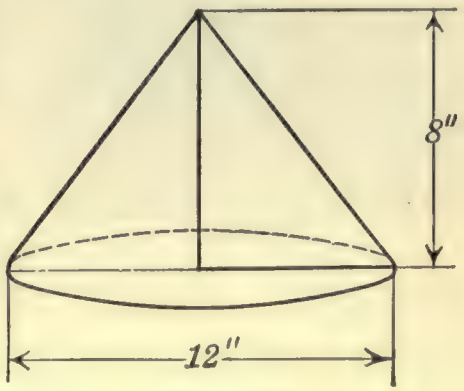


FIG. 62.

12. Find the weight of a steel cone 6″ in diameter and 5″ high, to the nearest tenth of a pound. The density of steel is .28 lb. per cu. in.

13. Find the weight of a cast iron cone 5″ in diameter and $8\frac{1}{2}$″ high, to the nearest tenth of a pound. The density of cast iron is .26 lb. per cu. in.

14. Draw a semicircle (half circle) having a 4″ diameter. Cut it out and fit the radii together so as to form the curved surface of a cone.

(*a*) How many inches are there in the arc of the semicircle?

(*b*) How many inches will there be in the circumference of the base of the cone formed?

(*c*) Find the radius of the base of the cone.

(*d*) How many square inches are there in the curved surface of the cone?

IV. Spheres

§ 78. A ***sphere*** is a round solid inclosed by a surface, all points of which are equidistant from a point within called the center. The line from the center to any point in the surface is the *radius*. The line through the center from surface to surface is the *diameter*.

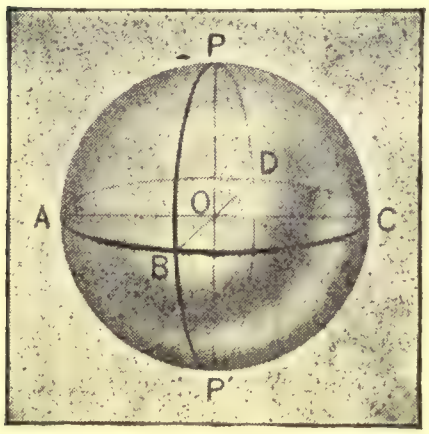


Fig. 63.

§ 79. Surface of a Sphere. The area of the surface of a sphere is equal to 4π times the square of its radius. The rule is expressed by the formula:

$$S = 4\pi r^2,$$

where

S = the surface of a sphere (in square units),

r = the radius (in linear units).

§ 80. Volume of a Sphere. The volume of a sphere is equal to $\frac{4}{3}\pi$ times the cube of the radius. This rule is expressed by the formula:

$$V = \tfrac{4}{3}\pi r^3,$$

where

V = the volume of a sphere (in cubic units),

r = the radius (in linear units).

Example. Find the volume of a sphere whose radius is 8″.

Solution.

$V = \frac{4}{3}\pi r^3$

① $V = \frac{4}{3} \times 3.14 \times 8^3$ (*Est.* $V = 2000$ cu. in.)

② $V = 2144$ *Ans.* $V = 2144$ cu. in.

EXERCISES

Copy the following table and fill in the required values.

FORMULAS: $S = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$.

	r	d	EST. S	S	EST. V	V
1.	5″		315 sq. in.		525 cu. in.	
2.	6″					
3.	12″					
4.	4.5′					
5.		44″				
6.		2′ 6″				
7.		3′				
8.	$2\frac{1}{2}$″					

9. A gilded dome is in the form of a hemisphere whose diameter is 40 feet. How many square feet are there in its surface? (A hemisphere is a half sphere.)

10. The density of steel is .28 lb. per cubic inch. Find the weight of a steel ball 10″ in diameter.

11. A croquet ball is 4″ in diameter. Find the weight of 10 balls. (The density of the wood is 49 lb. per cu. ft.)

12. A bowl in the form of a hemisphere is 8″ in diameter (inside). Find how many cubic inches of water it will hold. How many pints (to the nearest tenth) will it hold?

13. Compare the volume of a sphere whose radius is 2″ with that of a sphere whose radius is 4″. Compare their surface areas.

14. Compare the surface area, and the volume, of a sphere with those of a sphere whose radius is twice as great; three times as great.

15. A cone, a sphere, and a cylinder have the same diameter 10″ and height 10″ (Fig. 64).

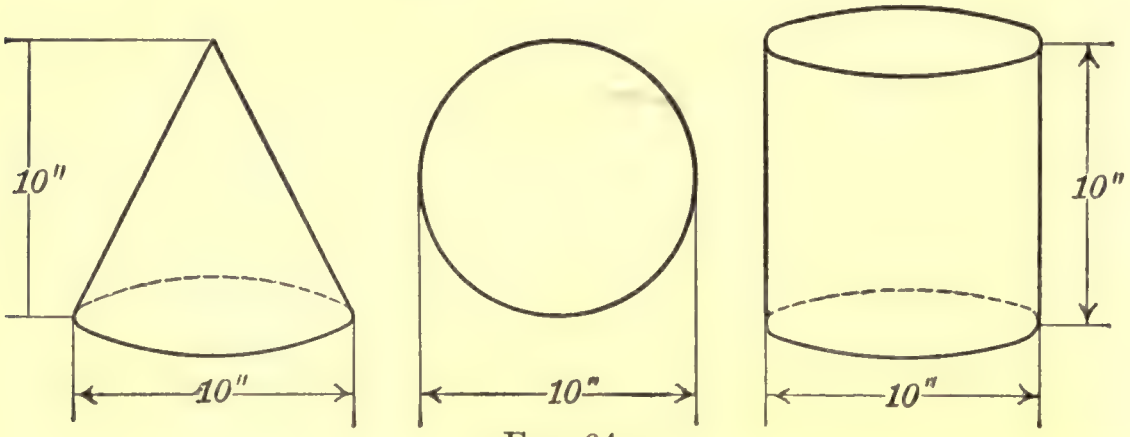


FIG. 64.

(*a*) Find the volume of each, and arrange them in order of size.

(*b*) Find the area of the entire surface of each, and arrange them in order of size.

16. The edge of a cube is 5″. The diameter of a sphere is 5″.

(*a*) Find the entire surface of the cube.

(*b*) Find the surface of the sphere.

(*c*) The entire surface of the cube is what per cent larger than that of the sphere?

(*d*) Find the volume of the cube.

(*e*) Find the volume of the sphere.

(*f*) The volume of the cube is what per cent larger than that of the sphere?

CHAPTER VIII

LINEAR EQUATIONS

I. Axioms of Addition, Subtraction, and Multiplication

§ 81. Group A. Solution by the Axiom of Subtraction.

Example 1. The length of a room is 3 feet more than twice the width. The length is 15 feet. Find the width.

Solution. Let w = the width of the room, then,

$2w + 3$ = the length.

The equation is:

① $\qquad 2w + 3 = 15$

If you take 3 away from each member of ①, then $2w$ is left in the first member and 12 is left in the second member, hence the equation takes the form

② $\qquad 2w = 12$

③ $\qquad w = 6 \qquad$ ② ÷ 2

Check. Substitute 6 feet for the width of the room in the statement of the given example.

Ans. The width of the room is 6 feet.

Briefly this solution may be written:

① $\qquad 2w + 3 = 15$

② $\qquad 2w + 3 - 3 = 15 - 3 \qquad$ ① − 3

③ $\qquad 2w = 12 \qquad$ ② ≡

④ $\qquad w = 6 \qquad$ ③ ÷ 2

[The symbol, ① − 3, means that 3 is subtracted from each member of equation ①.]

EXAMPLE 2. The perimeter of a rectangle is 48″. Its height is 10″. Find its base.

SOLUTION. The formula is: $p = 2\,b + 2\,h$.

Placing 48 for p and 10 for h,

①	$48 = 2\,b + 20$	
②	$48 - 20 = 2\,b + 20 - 20$	① − 20
③	$28 = 2\,b$	② ≡
④	$14 = b$	③ ÷ 2

CHECK. Substitute 14″ for the base of the rectangle in the statement of the given example.

Ans. The base of the rectangle is 14″.

EXAMPLE 3. Solve the equation $x + 14 = 27$, and check the answer.

SOLUTION.

①	$x + 14 = 27$	
②	$x + 14 - 14 = 27 - 14$	① − 14
③	$x = 13$	② ≡

CHECK. $13 + 14 = 27$ *Ans.* $x = 13$.

NOTE. Equation ② may be omitted, then ③ can be obtained directly by subtracting 14 from each member of ①.

EXAMPLE 4. Solve the equation $4\,a + 9 = 25$, and check the answer.

SOLUTION.

①	$4\,a + 9 = 25$	
②	$4\,a + 9 - 9 = 25 - 9$	① − 9
③	$4\,a = 16$	② ≡
④	$a = 4$	③ ÷ 4

CHECK. $(4 \times 4) + 9 \stackrel{?}{=} 25$

$25 = 25$ *Ans.* $a = 4$.

[Here again ③ may be obtained directly from ①.]

NOTE. In solving examples 1–4, we have used the following axiom.

AXIOM. *If the same number is subtracted from equal numbers, the remainders are equal.*

EXERCISES

Solve each of the following equations, and check each answer.

1. $x + 3 = 18$

2. $a + 7 = 12$

3. $x + 2 = 18$

4. $x + 8 = 11$

5. $14 = m + 5$

6. $28 = a + 15$

7. $1428 = b + 356$

8. $d + .9 = 1.7$

9. $12 = d + 8.7$

10. $10.7 = r + 3$

11. $2y + 5 = 7$

12. $10 = 2r + 4$

13. $3b + 4 = 16$

14. $15 = 10b + 5$

15. $2p + 12 = 17$

16. $6f + 3 = 18$

17. $63 = 4h + 19$

18. $38 = 2c + 9$

19. $5.5h + 4.2 = 20.7$

20. $98.4 = 3.5n + 17.9$

21. One number is 5 greater than another. Their sum is 47. Find the numbers.

SOLUTION. Let n = the smaller number, then,

$n + 5$ = the larger number.

①	$n + n + 5 = 47$	
②	$2n + 5 = 47$	① ≡
③	$2n = 42$	② − 5
④	$n = 21$	③ ÷ 2
⑤	$n + 5 = 26$	

CHECK. $21 + 26 = 47$

Ans. The two numbers are 21 and 26.

22. The sum of two numbers is 28, their difference is 8. What are the numbers? (Let them be n and $n+8$. Why?)

23. The sum of two numbers is 75, their difference is 11. What are the numbers?

24. The sum of two numbers is 128, the larger exceeds the smaller by 20. What are the numbers?

25. The sum of two numbers is 49, the larger exceeds the smaller by 12. What are the numbers?

26. The larger of two supplementary angles exceeds twice the smaller by 12°. What are the angles?

(Represent the angles by A and $2A + 12$. Why?)

27. In a company of 30 people a motion is carried by a majority of 6. How many voted for the measure? (Let n = the number voting against the measure.)

28. Three buckets of water fill a 12-gallon tub all but 6 qt. How much does the bucket hold?

29. A bag, weighing 3 oz., contains 35 marbles. The total weight is 38 oz. What is the weight of one marble?

30. A boy starts with a wage of \$30 a month and is to be given an increase of \$2.50 each successive month. In how many months will he be getting \$65?

§ 82. Group B. Solution by the Axiom of Addition.

EXERCISES

Complete each of the following statements.

1. $m - 5 + 5 = ?$

2. $k - 3 + 3 = ?$

3. $y - 7 + 7 = ?$

4. $a - 15 + 15 = ?$

5. $a - 1.2 + 1.2 = ?$

6. $2x - 13 + ? = 2x.$

7. $5a - 7 + ? = 5a.$

8. $8x - 3.5 + ? = 8x.$

9. $12m - 12.2 + ? = 12m$

10. $5.3b - 1.6 + ? = 5.3b$

11. $3 - 4x + 4x = ?$

12. $12 - 9a + 9a = ?$

13. $5 - 3m + ? = 5$

14. $15 - 4p + ? = 15$

15. $4.5 - 3.7y + ? = 4.5$

16. $12.5 - 4.2n + ? = 12.5$

EXAMPLE 1. My brother's age is 25 years less than twice my age. He is now 43 years old. How old am I?

SOLUTION. Let y = my age in years, then,

$2y - 25$ = my brother's age.

①	$2y - 25 = 43$	

43 is 25 less than $2y$, so to get $2y$ you will need to add 25 to each member of ①, giving 68 for the value of $2y$. The solution is then written as follows:

①	$2y - 25 = 43$	
②	$2y - 25 + 25 = 43 + 25$	① + 25
③	$2y = 68$	② ≡
④	$y = 34$	③ ÷ 2

CHECK. Substitute 34 years for my age in the statement of the given example.

Ans. I am 34 years old.

[The symbol, ① + 25, means that 25 is added to each member of ①.]

NOTE. In ① you had a *shortage* of 25, so you added 25 to get rid of this shortage; that is, $2y - 25 + 25 = 2y$.

EXAMPLE 2. The larger of two angles is 5° less than 4 times the smaller. If the larger angle is 115°, find the smaller.

SOLUTION. Let A = the smaller angle, then,

$4A - 5$ = the larger angle.

①	$4A - 5 = 115$	
②	$4A - 5 + 5 = 115 + 5$	① + 5
③	$4A = 120$	② ≡
④	$A = 30$	③ ÷ 4

CHECK. Substitute 30° for the smaller angle in the statement of the given example.

Ans. The smaller angle is 30°.

EXAMPLE 3. Solve the equation $y - 8 = 15$, and check the answer.

SOLUTION.

①	$y - 8 = 15$	
②	$y - 8 + 8 = 15 + 8$	① + 8
③	$y = 23$	② ≡
CHECK.	$23 - 8 = 15$	

Ans. $y = 23$.

NOTE. Equation ② may be omitted and ③ obtained directly from ① by adding 8 to each member of ①.

EXAMPLE 4. Solve the equation $18 = 5r - 2$, and check the answer.

SOLUTION.

①	$18 = 5r - 2$	
②	$18 + 2 = 5r - 2 + 2$	① + 2
③	$20 = 5r$	② ≡
④	$4 = r$	③ ÷ 5
CHECK.	$18 \stackrel{?}{=} (5 \times 4) - 2$	
	$18 = 18$	

Ans. $r = 4$.

[Here again ③ may be obtained directly from ①.]

NOTE. In solving examples 1–4, we have used the following axiom for making up the shortages.

AXIOM. *If the same number is added to equal numbers, the sums are equal.*

EXERCISES

Solve each of the following equations, and check each answer.

1. $x - 3 = 15$

2. $y - 4 = 6$

3. $10 = y - 5$

4. $4 = z - 3$

5. $m - 5.4 = 3.6$

6. $10 = x - 3.7$

7. $b - 3\frac{1}{2} = 7\frac{3}{4}$

8. $a - 3\frac{1}{4} = 5\frac{3}{8}$

9. $7.45 = m - 2.2$

10. $6\frac{3}{4} = x - 2.4$

11. $3x - 4 = 8$

12. $2a - 4 = 8$

13. $5x - 3 = 7$

14. $5a - 7 = 9$

15. $4b - 6 = 17$

16. $44 = 7c - 8.5$

17. $3d - 4.2 = 3.6$

18. $8k - 6 = 46$

19. $5.3x - 8.54 = 7.36$

20. $3\frac{1}{2}x - 6\frac{1}{4} = 18\frac{1}{4}$

21. In 1915 the number of representatives in the United States was 435. This number was 45 less than five times the number of senators. How many senators were there?

22. In 1915 New York had one less than four times as many representatives in Congress as California. New York had 43 representatives; how many had California?

23. The smaller of two supplementary angles is 18° less than the larger. Find the two angles.

24. The larger of two supplementary angles is 30° less than 6 times the smaller. Find the two angles. (Represent the two angles by A and $6A - 30$. Why?)

25. The larger of two supplementary angles is 75° less than 4 times the smaller. Find the two angles.

26. The average rate per hour of a boy on a bicycle with a motor attachment is 4 miles less than three times his rate without the attachment. His average rate with the attachment is 44 miles per hour. Find his rate without the attachment.

27. The population (nearest thousand) of New York City in 1914 was 1,595,000 less than 3 times the population of Chicago. The sum of their populations was 7,977,000. Find the population of each.

§ 83. Group C. Solution by the Axioms of Addition and Subtraction.

EXAMPLE 1. Solve the equation $3x + 2 = x + 18$, and check the answer.

SOLUTION.

Here there are four terms. The first step in solving this equation is to get rid of the *smaller unknown* term.

①	$3x + 4 = x + 18$	
②	$3x - x + 4 = x - x + 18$	① $- x$
③	$2x + 4 = 18$	② $\equiv$
④	$2x = 14$	③ $- 4$
⑤	$x = 7$	④ $\div 2$

CHECK. $(3 \times 7) + 4 \stackrel{?}{=} 7 + 18$

$21 + 4 = 25$

Ans. $x = 7$.

NOTE. 7 is the ***root*** of the equation, $3x + 4 = x + 18$.

The *root* of an equation is that value of the unknown which satisfies the equation; that is, makes both members have the same value.

EXAMPLE 2. Solve the equation $5n - 6 = 2n + 12$, and check the answer.

SOLUTION.

Here there is a shortage to be made up *before* getting rid of the smaller unknown term.

①	$5n - 6 = 2n + 12$	
②	$5n = 2n + 18$	① $+ 6$
③	$3n = 18$	② $- 2n$
④	$n = 6$	③ $\div 3$

CHECK. $(5 \times 6) - 6 \stackrel{?}{=} (2 \times 6) + 12$

$30 - 6 \stackrel{?}{=} 12 + 12$

$24 = 24$

Ans. $n = 6$.

Is 6 the root of the equation, $5n - 6 = 2n + 9$? How do you know?

EXAMPLE 3. Solve the equation

$$b + 3 = 3b - 4,$$

and check the answer.

SOLUTION.

①	$b + 3 = 3b - 4$	
②	$b + 7 = 3b$	① + 4
③	$7 = 2b$	② − b
④	$3.5 = b$	③ ÷ 2

CHECK. $3.5 + 3 \stackrel{?}{=} (3 \times 3.5) - 4$

$6.5 = 10.5 - 4$

Ans. $b = 3.5$.

What is the root of the equation,

$$b + 3 = 3b - 4?$$

Why?

In general, the student should thus check every answer obtained.

EXERCISES

Solve each of the following equations, and check each answer.

1. $5x - 2 = x + 22$
2. $5n - 7 = 2n + 5$
3. $3k + 2 = k + 7$
4. $10k + 4 = 4k + 16$
5. $5r + 7 = 9r + 3$
6. $4a + 13 = 8a - 11$
7. $2a + 5 = 7a - 40$
8. $3r + 6 = 7r - 14$
9. $2r + 15 = 5r + 3$
10. $1.8m - 3 = 1.2m + 21$
11. $15t + 7 = 13t + 11$
12. $5w + 3 = 9w - 21$
13. $4a - 9 = 3a + 15$
14. $5a + 6 = 7a - 6$
15. $3a + 5 = 2a + 5$
16. $5a - 6 = 3a + 16$
17. $7w - 2 = 4w + 17$
18. $4x + 13 = 9x + 3$
19. $6h + 190 = 12h + 13$
20. $4m + 3 = 9m - 19$

21. $12\,a + 4 = 84 - 8\,a$

SOLUTION.

First make up the shortage by adding $8\,a$.

①	$12\,a + 4 = 84 - 8\,a$	
②	$20\,a + 4 = 84$	① $+ \ 8\,a$
③	$20\,a = 80$	② $- \ 4$
④	$a = 4$	③ $\div \ 20$

CHECK. $(12 \times 4) + 4 \stackrel{?}{=} 84 - (8 \times 4)$

$48 + 4 \stackrel{?}{=} 84 - 32$

$52 = 52$

Ans. $a = 4$.

22. $3\,x + 2 = 22 - 2\,x$

23. $15\,m + 5 = 57 - 11\,m$

24. $7\,k - 4 = 16 - 3\,k$

25. $6\,y - 10 = 15 - 4\,y$

26. $15 - 2\,y = 3\,y + 5$

27. $25 - y = 7\,y - 7$

28. $6\,m - 7 = 3\,m + 14$

29. $5\,a + 17 = 13\,a - 3$

30. $5\,a - 6 = 40 - 3\,a$

31. Five times a certain number increased by 4 is equal to four times the number increased by 10. Find the number.

32. Eight times a certain number diminished by 6 is equal to five times the number increased by 27. Find the number.

33. The ratio of two numbers is 5 to 3. 8 less than the larger number is 4 more than the smaller. Find the numbers. (Let the numbers be $5\,n$ and $3\,n$.)

Ans. The two numbers are 30 and 18.

34. The ratio of two numbers is 7 to 3. 17 less than the larger number is 15 more than the smaller. Find the numbers.

35. The length and width of a room have a ratio of 8 to 5. The room would be square if it were 4 feet shorter and 2 feet wider. Find the length and width of the room. What is its area? What would be the area of the square room?

§ 84. Group D. Solution by the Axiom of Multiplication.

EXAMPLE 1. Solve the equation

$$\frac{3\,x}{5} + \frac{x}{3} = 28,$$

and check the answer.

SOLUTION.

The first step in solving fractional equations is to get rid of the fractions. For this equation 15 is the multiplier that will get rid of both denominators 5 and 3.

①	$\frac{3\,x}{5} + \frac{x}{3} = 28$	
②	$\overset{3}{\cancel{15}}\left(\frac{3\,x}{\cancel{5}}\right) + \overset{5}{\cancel{15}}\left(\frac{x}{\cancel{3}}\right) = 15 \times 28$	① × 15
③	$9\,x + 5\,x = 420$	② ≡
④	$14\,x = 420$	③ ≡
⑤	$x = 30$	④ ÷ 14

CHECK. $\frac{3 \times 30}{5} + \frac{30}{3} \overset{?}{=} 28$

$$18 + 10 = 28$$

Ans. $x = 30$.

Why is 30 the root of the equation,

$$\frac{3\,x}{5} + \frac{x}{3} = 28?$$

EXAMPLE 2. Solve the equation

$$\frac{10\,m}{3} - 4 = \frac{3\,m}{2} + 18,$$

and check the answer.

SOLUTION.

To get rid of the denominators 3 and 2, use 6 for the multiplier. Why?

①	$\frac{10\,m}{3} - 4 = \frac{3\,m}{2} + 18$	
②	$\cancel{6}^{2}\left(\frac{10\,m}{\cancel{3}}\right) - 6 \times 4 = \cancel{6}^{3}\left(\frac{3\,m}{\cancel{2}}\right) + 6 \times 18$	① × 6
③	$20\,m - 24 = 9\,m + 108$	② ≡
④	$20\,m = 9\,m + 132$	③ + 24
⑤	$11\,m = 132$	④ − 9 m
⑥	$m = 12$	⑤ ÷ 11

CHECK.

$$\frac{10 \times 12}{3} - 4 \stackrel{?}{=} \frac{3 \times 12}{2} + 18$$

$$40 - 4 \stackrel{?}{=} 18 + 18$$

$$36 = 36.$$

Ans. $m = 12$.

Summary for the Solution of a Linear Equation.

(1) If the equation contains fractions, multiply each member by the *least common multiple* of all the denominators.

(2) Collect all unknown terms in one member, and all known terms in the other member, by adding and subtracting the necessary terms. *Shortages should be made up first.*

(3) Divide each member by the numerical factor of the unknown term.

EXERCISES

Find the root of each of the following equations, and check each root.

1. $\frac{x}{6} = \frac{23}{3}$

2. $\frac{x}{3} + \frac{x}{2} = 15$

3. $\frac{2\,m}{3} = 8$

4. $\frac{2\,w}{3} = \frac{4}{5}$

5. $\frac{3\,b}{5} = \frac{2}{15}$

6. $\frac{5\,a}{3} = \frac{15}{2}$

7. $7 = \frac{2\,a}{3}$

8. $\frac{5\,x}{4} = .4$

9. $\frac{d}{4} = \frac{3}{100}$

10. $\frac{5}{6} = \frac{2\,w}{15}$

11. $n + \frac{n}{2} + \frac{n}{3} = 88$

12. $\frac{n}{2} + \frac{n}{3} - \frac{n}{4} + \frac{n}{10} = 82$

13. $\frac{4\,x}{3} + \frac{2\,x}{5} = \frac{26}{5}$

14. $\frac{4}{3}\,x - \frac{3}{5}\,x = 4\frac{2}{5}$

(Suggestion: $\frac{4\,x}{3} - \frac{3\,x}{5} = \frac{22}{5}$.)

15. $\frac{3}{4}\,x + 1\frac{1}{2} = \frac{7}{8}\,x$

16. $\frac{m}{3} - \frac{m}{7} = \frac{m}{6} + 1$

17. $\frac{y}{2} - \frac{y}{5} = \frac{y}{2} - 3$

18. $\frac{2\,y}{3} - \frac{3}{5} = 0$

19. $\frac{12\,p}{5} = 14 + p$

20. $\frac{3\,a}{5} + 4 = \frac{2\,a}{3} - 1$

21. $\frac{b + 12}{2} = b + 5$

22. $\frac{3\,y + 10}{5} = y + 1$

23. One half of a certain number plus one third of the number is 10. Find the number.

24. The difference between one third of a number and one fifth of the number is 6. Find the number.

25. How high is a tree that casts a shadow 40 feet long, if a 6-foot vertical post casts a shadow 8 feet long at the same time? (See Fig. 65, scale: 20′ to 1″.)

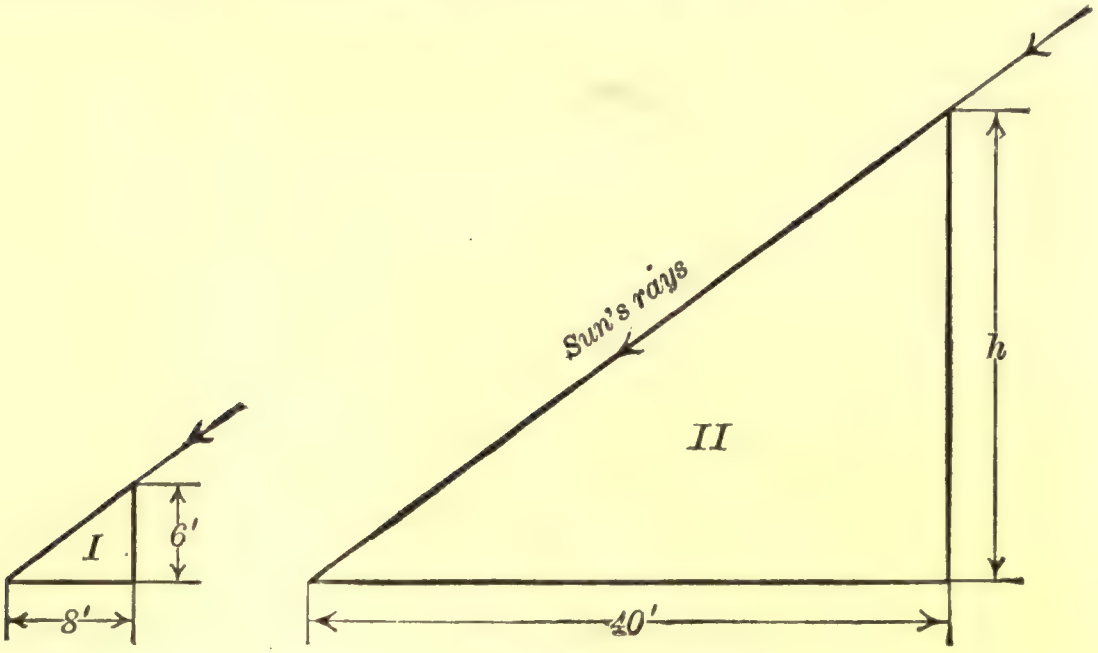


FIG. 65.

NOTE. The sun's rays form two triangles, that have the same shape. Triangles that have the same shape are called ***similar triangles***. Their corresponding sides have the same ratio to each other; that is,

$$\frac{\text{the height of } \triangle \text{ II}}{\text{the height of } \triangle \text{ I}} = \frac{\text{the base of } \triangle \text{ II}}{\text{the base of } \triangle \text{ I}}.$$

For this example the equation is:

$$\frac{h}{6} = \frac{40}{8}.$$

26. Find the height of a flagstaff which casts a shadow of 77 feet on level ground, if at the same time a ten-foot vertical rod casts a seven-foot shadow.

27. Find the height of your school building, using the method in exercise 26.

28. A map is drawn to the scale of 40 miles to 1 inch. On this map two cities are $2\frac{1}{2}$ in. apart. How many miles are there between the cities?

$\left(\text{The equation is: } \frac{m}{40} = \frac{2.5}{1}.\right)$

29. On a city map the scale is $\frac{1}{2}$ mile to one inch. Find how far apart two buildings are, if the distance on the map is $2\frac{1}{4}$ in.

30. On a map of the United States the scale is 300 mi. to the inch. What is the cross-country distance between two cities, if the distance between them on the map is 1.7 in.?

31. Draw a triangle having angles of 50°, 60°, and 70°. Draw another larger triangle having the same three angles. Letter them as in Fig. 66. These triangles are similar.

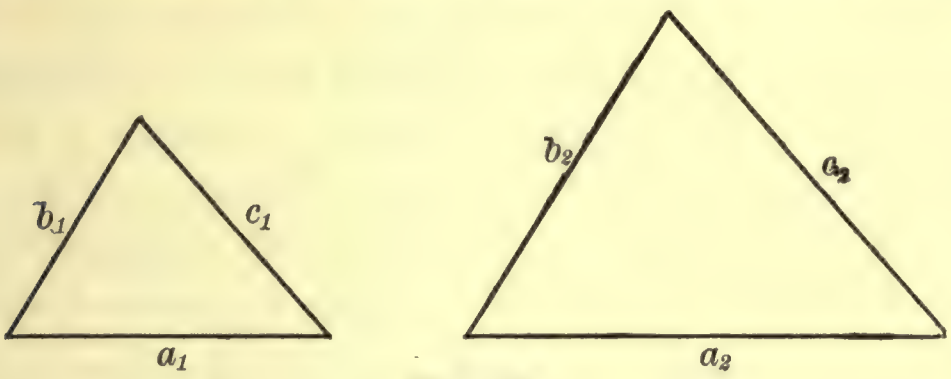


FIG. 66.

(*a*) Measure a_1 and a_2. $\frac{a_1}{a_2} = ?$

(*b*) Measure b_1 and b_2. $\frac{b_1}{b_2} = ?$

(*c*) Measure c_1 and c_2. $\frac{c_1}{c_2} = ?$

(*d*) $\frac{a_1}{a_2} \stackrel{?}{=} \frac{b_1}{b_2}$. $\frac{a_1}{a_2} \stackrel{?}{=} \frac{c_1}{c_2}$.

NOTE. The equality of two ratios is called a *proportion*, hence the sides of similar triangles are in *proportion*.

32. The sides of one triangle are $a_2 = 10''$, $b_2 = 12''$, and $c_2 = 15''$. In a similar triangle $a_1 = 8''$. Find b_1 and c_1.

$\left(\text{The equations are: } \frac{8}{10} = \frac{b_1}{12} \text{ and } \frac{8}{10} = \frac{c_1}{15}.\right)$

33. The sides of a triangle are $4''$, $5''$, and $7''$. The shortest side of a similar triangle is $3''$. Find the other two sides.

34. The sides of a triangle are $3.9''$, $4.2''$, and $5.7''$. The shortest side of a similar triangle is $1.9''$. Find the other two sides.

35. If 2 lb. of beef are needed to make beef stew for 9 people, how much beef would be needed to make a stew for 30?

36. The directions for making a substitute for maple syrup from a patent liquid called "Mapleine" are as follows: For one gallon of syrup, use 7 lb. granulated sugar, 4 pints hot water, and two tablespoonfuls mapleine. How many of each will be needed to make 5 quarts of syrup?

37. If two quarts of ice cream are required for 10 boys, how many quarts should be made for a camp of 25 boys?

38. The earth is about 8000 miles in diameter; Mt. Everest is about 29,000 feet high; the greatest ocean depth known is about 32,000 feet. On a model of the earth the size of an orange, $4''$ in diameter,

(*a*) How high would Mt. Everest appear?

(*b*) How deep would the greatest ocean depth appear?

II. Removal of Parentheses

§ 85. Group E. Solution by the Removal of Parentheses.

EXERCISES

1. (*a*) $5(3 + 4) = 5 \times 7 = 35.$

(*b*) $5(3 + 4) = 15 + 20 = 35.$

In (*a*) you first add 3 and 4, then multiply by 5.

In (*b*) you first multiply 3 by 5 and 4 by 5, then add the results.

2. $3(2 + 7) = ?$ Add first.

$3(2 + 7) = ?$ Multiply first.

3. Find the results for each of the following in two ways:

(*a*) $4(3 + 2)$ (*d*) $3(7 + 4)$

(*b*) $7(4 + 3)$ (*e*) $6(5 + 2)$

(*c*) $10(5 + 2)$ (*f*) $7(3 + 8)$

4. Find the results for each of the following in two ways:

(*a*) $5(7 - 2)$ Method: $5(7 - 2) = 5 \times 5 = 25$

or $5(7 - 2) = 35 - 10 = 25$

(*b*) $6(8 - 3)$ (*d*) $12(5 - 2)$

(*c*) $4(12 - 7)$ (*e*) $7(8 - 3)$

5. $4(a + 3) = ?$

This result can be obtained in only one way, that way being to multiply a and 3 by 4; thus

$$4(a + 3) = 4a + 12.$$

6. $7(x - 2) = ?$

$$7(x - 2) = 7x - 14.$$

7. Expand the following expressions by multiplication:

(*a*) $5(m + 3)$ (*d*) $7(2 + k)$

(*b*) $8(a + 4)$ (*e*) $10(7 + y)$

(*c*) $12(m + 3)$ (*f*) $7(5 + n)$

8. Expand the following expressions by multiplication:

(*a*) $3(b - 2)$ (*d*) $8(10 - h)$

(*b*) $5(d - 12)$ (*e*) $12(4 - n)$

(*c*) $6(7 - a)$ (*f*) $7(5 - x)$

9. Expand the following expressions by multiplication:

(*a*) $5(2a + 7)$ *Ans.* $10a + 35$. (*d*) $2.5(4b + 3)$

(*b*) $3(4a - 2)$ (*e*) $5.2(3.5 - 8m)$

(*c*) $4(5 + 2a)$ (*f*) $7.5(2.4 - 5x)$

10. Solve the equation $5(2a - 3) = 25$, and check the answer.

SOLUTION.

①	$5(2a - 3) = 25$	
②	$10a - 15 = 25$	① ≡
③	$10a = 40$	② + 15
④	$a = 4$	③ ÷ 10

CHECK. $5[(2 \times 4) - 3] \stackrel{?}{=} 25$

$5(8 - 3) \stackrel{?}{=} 25$

$5 \times 5 = 25$ *Ans.* $a = 4$.

When checking, first get the value of the quantity within the parentheses.

Solve each of the following equations, and check each answer.

11. $2(b + 3) = 14$ **16.** $15 = 3(k - 1)$

12. $4(m - 2) = 20$ **17.** $20 = 8(a - 3)$

13. $5(w - 4) = 35$ **18.** $12 = 3(y + 1)$

14. $7(c - 1) = 35$ **19.** $15(c - 7) = 45$

15. $3(d + 7) = 45$ **20.** $2(k + 4) = 23$

21. $13(w - 3) = 5(w + 5)$

SOLUTION.

①	$13(w - 3) = 5(w + 5)$	
②	$13\,w - 39 = 5\,w + 25$	① ≡
③	$13\,w = 5\,w + 64$	② + 39
④	$8\,w = 64$	③ − 5 w
⑤	$w = 8$	④ ÷ 8

CHECK. $13(8 - 3) \stackrel{?}{=} 5(8 + 5)$

$13 \times 5 \stackrel{?}{=} 5 \times 13$

$65 = 65$

Ans. $w = 8$.

22. $5(t + 2) = 2(t + 14)$

23. $8(k + 1) = 4(k + 9)$

24. $4(m - 2) = 3(m + 1)$

25. $6(y - 7) = 4(y + 2)$

26. $5(a - 3) = 2(a + 3)$

27. $8(n + 5) = 2(3\,n + 26)$

28. $4(3\,a - 2) = 3(2\,a + 4)$

29. $5(2\,a + 1) = 3(4 - a)$

30. $3(5 - 2\,x) = 5(x + 2)$

31. $4(8 - x) = 6(x + 3)$

32. The sum of two numbers is 98. Twice the greater equals 14 plus five times the less. Find the two numbers.

SOLUTION. Let n = the smaller number, then

$98 - n$ = the larger number.

The equation is:

①	$2(98 - n) = 14 + 5\,n$	
②	$196 - 2\,n = 14 + 5\,n$	① ≡
③	$196 = 14 + 7\,n$	② + 2 n
④	$182 = 7\,n$	③ − 14
⑤	$26 = n$	④ ÷ 7
⑥	$98 - n = 72$	

CHECK. Substitute 26 and 72 for the two numbers in the statement of the given example.

Ans. The two numbers are 26 and 72.

33. The sum of two numbers is 15. 6 times the smaller equals 3 times the larger. Find the numbers.

(Represent the numbers by n and $15 - n$.)

34. The sum of two numbers is 28. 5 times the larger equals 9 times the smaller. Find the numbers.

35. The difference between two numbers is 5. Four times the larger equals five times the smaller. Find the numbers.

(Represent the numbers by n and $n + 5$.)

36. The difference between two numbers is 7. If 8 is added to 4 times the smaller, the result equals 3 times the larger. Find the numbers.

37. The value of 20 coins consisting of nickels and dimes is \$1.40. How many are there of each?

(Represent the number of dimes by d and the number of nickels by $20 - d$. The equation is: $10\,d + 5(20 - d) = 140$. Why?)

38. The value of 25 coins consisting of nickels and pennies is 85 cents. How many are there of each?

(Let n = the number of nickels.)

39. How many quarts of water must be mixed with 40 quarts of alcohol, 80 % pure, to make a mixture 75 % pure?

(The equation is: $\frac{4}{5} \times 40 = \frac{3}{4}\,(40 + w)$. Why?)

40. A certain grade of listerine is 25 % alcohol. How many parts of water must be added to 100 parts of listerine to make it 10 % alcohol?

41. How many parts of water must be added to 100 parts of a solution that contains 70 % carbolic acid to make a 10 % solution? A 5 % solution?

§ 86. Group F. Solution by a Combination of Methods.

EXAMPLE 1. Solve the equation

$$\frac{2x+3}{5} = 7,$$

and check the answer.

SOLUTION.

①	$\frac{2x+3}{5} = 7$	
②	$\cancel{5}\left(\frac{2x+3}{\cancel{5}}\right) = 5 \times 7$	① × 5
③	$2x + 3 = 35$	② ≡
④	$2x = 32$	③ − 3
⑤	$x = 16$	④ ÷ 2

CHECK.

$$\frac{(2 \times 16) + 3}{5} \stackrel{?}{=} 7$$

$$\tfrac{35}{5} = 7$$

Ans. $x = 16$.

EXAMPLE 2. Solve the equation

$$\frac{m+8}{3} = \frac{3m-4}{5},$$

and check the answer.

SOLUTION.

①	$\frac{m+8}{3} = \frac{3m-4}{5}$	
②	$\cancel{15}^{5}\left(\frac{m+8}{\cancel{3}}\right) = \cancel{15}^{3}\left(\frac{3m-4}{\cancel{5}}\right)$	① × 15
③	$5(m+8) = 3(3m-4)$	② ≡
④	$5m + 40 = 9m - 12$	③ ≡
⑤	$5m + 52 = 9m$	④ + 12
⑥	$52 = 4m$	⑤ − 5 m
⑦	$13 = m$	⑥ ÷ 4

Ans. $m = 13$.

EXERCISES

Find the root of each of the following equations, and check each root.

1. $\frac{x+1}{2} = 2$

2. $\frac{y-4}{5} = 3$

3. $\frac{3m+1}{2} = 5$

4. $\frac{2a-1}{3} = 7$

5. $\frac{2p-5}{3} = 6$

6. $\frac{3a-5}{4} = 7$

7. $\frac{5a+2}{3} = 16$

8. $\frac{5a-1}{12} = 6$

9. $6 = \frac{4b+2}{3}$

10. $5 = \frac{6b-1}{4}$

11. $\frac{2a-5}{6} = \frac{5}{2}$

(Multiply by 6.)

12. $\frac{3x+4}{10} = \frac{14}{5}$

13. $\frac{5b-3}{12} = \frac{5}{8}$

14. $\frac{a-2}{3} = \frac{a+4}{5}$

15. $\frac{2k-15}{3} = \frac{k+3}{4}$

16. $\frac{4a}{5} + \frac{3}{4} = \frac{2a+1}{2}$

17. $\frac{3x-5}{4} = \frac{x}{3} + 7$

18. $16\frac{1}{2} = \frac{2}{3}(m-12)$

(Multiply by 6.)

19. The sum of two numbers is 38. One tenth of the greater number equals one ninth of the less. Find the numbers.

$\left(\text{The equation is: } \frac{n}{9} = \frac{38-n}{10}.\right)$

20. The sum of two numbers is 66. One half of the smaller plus one seventh of the greater is 18. Find the numbers.

(Represent the smaller number by n.)

21. The difference between two numbers is 5. One third of the smaller equals one fourth of the larger. Find the numbers.

(Represent the smaller number by n.)

22. What number must be added to the numerator of $\frac{5}{12}$ in order that the resulting fraction shall be equal to $\frac{3}{4}$?

$\left(\text{The equation is: } \frac{n+5}{12} = \frac{3}{4}.\right)$

23. What number must be added to the numerator of the fraction $\frac{4}{9}$ to get a fraction equal to $\frac{2}{3}$?

24. What number must be added to the first term of the ratio $\frac{9}{16}$ to get a ratio equal to $\frac{3}{4}$?

$\left(\text{The equation is: } \frac{9+n}{16} = \frac{3}{4}.\right)$

25. What number must be subtracted from the first term of the ratio $\frac{25}{32}$ to get a ratio equal to $\frac{9}{16}$?

26. If the sum of a number and 7 is divided by 3, the quotient is 23. Find the number.

27. If the sum of 3 times a number and 5 is divided by 4, the quotient is 38. Find the number.

28. Divide the number 68 in two parts such that one half the smaller part shall equal $\frac{4}{9}$ of the larger part. (Let n = the larger part.)

29. The sum of the angles of a polygon is $(n - 2)\,180°$, where n is the number of sides. How many sides has a polygon if the sum of its angles is 540°?

30. How many sides has each of the following polygons:

(*a*) A polygon, the sum of whose angles is 1080°?

(*b*) A polygon, the sum of whose angles is 1440°?

(*c*) A polygon, the sum of whose angles is 1800°?

III. Simultaneous Equations

§ 87. Graphs of Linear Equations. On pages 116–122, Chapter V, you were required to represent graphically groups of related statistics, laws, etc. You tabulated pairs of numbers; such as, certain hours of the day, or, certain populations corresponding to certain years. Then you located points on squared paper to correspond to these pairs of numbers and joined these points forming a *graph*. You plotted a graph showing the relation between the weight of water and its volume. (See Ex. 8, page 122.) This relation could have been expressed by the formula, $W = 62.5\,V$. This equation contains two quantities, W and V. The value of W depends upon the value given to V. You tabulated several pairs of values for V and W before you *plotted* the *graph*.

Example 1. Find pairs of values for x and y that will satisfy the equation $y + x = 4$, and tabulate them.

Solution. If $x = +3$, then $y + x = 4$ becomes $y + 3 = 4$, or $y = 1$. (Why?)

If $x = +2$, then $y + 2 = 4$, $y = 2$.
If $x = +1$, then $y + 1 = 4$, $y = 3$.
If $x = 0$, then $y = 4$.
If $x = -1$, then $y - 1 = 4$, $y = 5$. (Why?)
If $x = -2$, then $y - 2 = 4$, $y = 6$.
If $x = -3$, then $y - 3 = 4$, $y = 7$.
If $x = -4$, then $y - 4 = 4$, $y = 8$.

These values are tabulated as follows:

x	$+3$	$+2$	$+1$	0	-1	-2	-3	-4
y	$+1$	$+2$	$+3$	$+4$	$+5$	$+6$	$+7$	$+8$

Among the values for y that were used were numbers with which you are not familiar, the value written -1, -2, -3, and -4. The sign $-$ has been used to show subtraction, the opposite of addition. If $+4$ means 4 more than zero, then -4 means 4 less than zero. This fact may be represented on a scale as in Fig. 67.

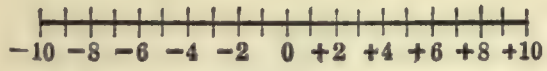


Fig. 67.

Or it may be convenient to represent it on a scale as in Fig. 68 as on a thermometer.

If these two scales are combined so as to cross at 0, then we have the combined scale shown in Fig. 69. You

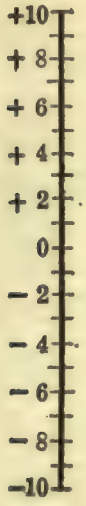


Fig. 68.

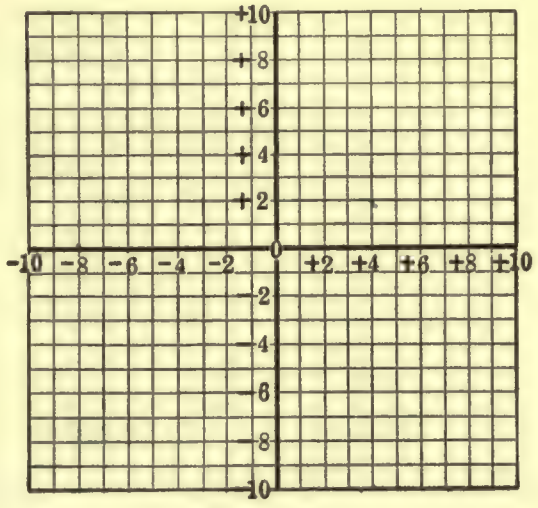


Fig. 69.

will see that when you plotted the graphs for statistics you used only *one* of the four sections here formed, the upper one on the right. If any of the temperatures recorded in the example on page 116 had been below zero you would have used both the upper and the lower sections on the right.

In Fig. 70, the horizontal line (*axis*) is the *x-axis;* that is, the axis along which the values of x are counted. The vertical axis is the *y-axis;* that is, the axis along which the values of y are counted.

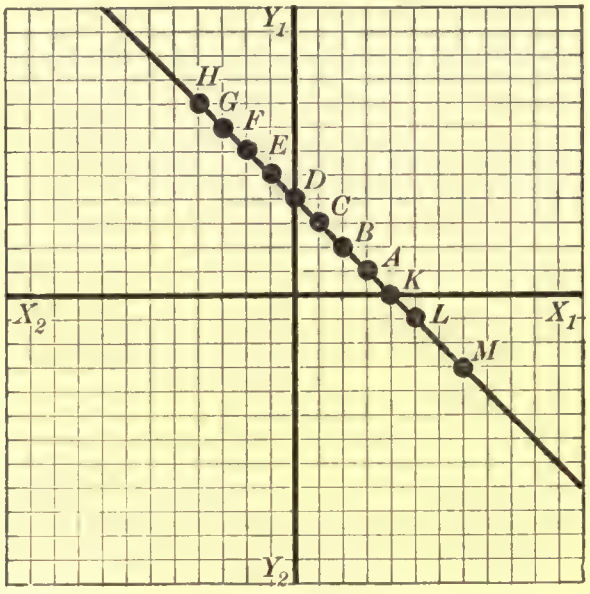


FIG. 70.

The points marked in Fig. 70 correspond to the pairs of values tabulated on page 196. Point A corresponds to the pair $x = +3$, $y = 1$ (3 to the right, 1 up). Point B corresponds to the pair $x = 2$, $y = 2$ (2 to the right and 2 up). Point C corresponds to the values $(+1, +3)$. Point D corresponds to the values $(0, +4)$ (neither to the right nor left, 4 up); etc. The straight line passing through A and H contains all the other points.

(*a*) Read the pair of values for the point K.
Do they satisfy the equation $y + x = 4$?

(*b*) Read the pair of values for the point L.
Do they satisfy the equation $y + x = 4$?

(*c*) Read the pair of values for the point M.
Do they satisfy the equation $y + x = 4$?

EXERCISES

1. Plot the graph of the equation

$$x + 2y = 8.$$

(*a*) The tabulation for certain pairs of values is:

x	-4	-2	0	$+2$	$+4$	$+6$	$+8$	$+10$
y	$+6$	$+5$	$+4$	$+3$	$+2$	$+1$	0	-1

(*b*) Locate these pairs of values on the squared paper. Do they lie in a straight line?

(*c*) Since two points determine a straight line, how many pairs of values are *necessary* for locating this graph?

The direction of the line is checked by locating a third point.

2. Plot the graph of the equation

$$3x + 2y = 10.$$

Find three pairs of values, using for x the values -2, 0, and $+2$.

3. Plot the graph of the equation

$$x + 3y = 12.$$

(Use for x the values -3, 0, and $+9$.)

NOTE. The four equations

$$y + x = 4,$$
$$x + 2y = 8,$$
$$3x + 2y = 10,$$
$$x + 3y = 12,$$

are called ***linear equations***, because their graphs are straight lines. Since all equations in this chapter are linear, their graphs are straight lines.

§ 88. Solution of Simultaneous Linear Equations by Graphs.

If the graphs of two linear equations intersect, one pair of values must satisfy both equations. Two such equations are ***simultaneous linear equations.***

EXAMPLE 1. Plot the graph of the equation $x + 2y = 8$. Also plot the graph of the equation $2x - y = 6$ on the same axes.

SOLUTION.

(I) $x + 2y = 8$.

x	0	2	6
y	4	?	?

(II) $2x - y = 6$.

x	3	?	?
y	0	4	8

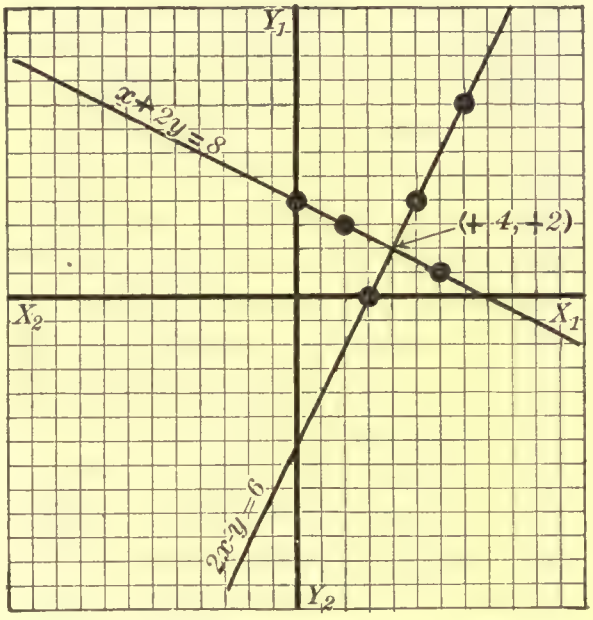


FIG. 71.

The graphs of these equations intersect at $(+4, +2)$; that is, $x = 4$, $y = 2$ is a pair of values that satisfies both equations. (See Fig. 71.)

CHECK.

$$\text{(I)} \quad \begin{cases} x + 2y = 8 \\ 4 + 4 = 8 \end{cases}$$

$$\text{(II)} \quad \begin{cases} 2x - y = 6 \\ 8 - 2 = 6 \end{cases}$$

Therefore $x = 4$, $y = 2$ is the solution for these two equations.

EXERCISES

Plot the graphs for each of the following pairs of equations. In any exercises where two equations are simultaneous, prove that the pair of values for the point of intersection of the two graphs satisfies both of the equations.

1. $\begin{cases} x + y = 6, \\ x + 2y = 8. \end{cases}$

2. $\begin{cases} x + y = 6, \\ x - y = 4. \end{cases}$

3. * $\begin{cases} 2x + 5y = 10, \\ 4x - 2y = 8. \end{cases}$

4. $\begin{cases} 2x + y = 5, \\ 4x + 2y = 8. \end{cases}$

5. $\begin{cases} 2x + 3y = 2, \\ 4x + 9y = 5. \end{cases}$

* NOTE. For the second equation in Ex. 3 use values 0, 4, and 8 for y.

In Ex. 5, you find that the two values of the point of intersection are fractional, so they can be obtained only approximately. The method given in the next section will show how to get the two values more accurately.

§ 89. Solution of Simultaneous Linear Equations by Elimination.

Simultaneous linear equations can be solved by so combining the two equations into one equation as to *eliminate*, or get rid of, one unknown, and then solving the resulting equation for the other unknown. The process of elimination to be used here is by *addition* or *subtraction*.

EXAMPLE 1. Solve the pair of simultaneous equations:

(I) $x + 3y = 8,$

(II) $2x - 3y = 7.$

SOLUTION.

If equation (II) is added to equation (I), member to member, the terms containing y will be eliminated.

③	$3x = 15$	(I)+(II)
④	$x = 5$	③ ÷ 3

Placing 5 for x in (I),

⑤	$5 + 3y = 8$	
⑥	$3y = 3$	⑤ − 5
⑦	$y = 1$	⑥ ÷ 3

CHECK.

(I) $5 + (3 \times 1) \stackrel{?}{=} 8$

$5 + 3 = 8$

(II) $(2 \times 5) - (3 \times 1) \stackrel{?}{=} 7$

$10 - 3 = 7$

Ans. $x = 5,\ y = 1.$

EXAMPLE 2. Solve the pair of simultaneous equations:

(I) $2x + 3y = 18,$

(II) $2x + y = 10.$

SOLUTION.

If (II) is subtracted from (I), member from member, then x will be eliminated.

③	$2y = 8$	(I) − (II)
④	$y = 4$	③ ÷ 2

Placing 4 for y in (I),

⑤	$2x + 12 = 18$	
⑥	$2x = 6$	⑤ − 12
⑦	$x = 3$	⑥ ÷ 2

CHECK.

$$\text{(I)} \quad (2 \times 3) + (3 \times 4) \stackrel{?}{=} 18$$
$$6 + 12 = 18$$
$$\text{(II)} \quad (2 \times 3) + 4 \stackrel{?}{=} 10$$
$$6 + 4 = 10$$

Ans. $x = 3,\ y = 4.$

To get x, could 4 have been placed for y in (II) instead of in (I)? Try it.

EXAMPLE 3. Solve the pair of simultaneous equations:

$$\text{(I)} \quad 2x - 3y = 4,$$
$$\text{(II)} \quad 3x + y = 17.$$

SOLUTION.

Could you eliminate x or y by adding (I) and (II)?

Could you eliminate x or y by subtracting (II) from (I)?

To eliminate y multiply (II) by 3 and add the resulting equation to (I).

(I)	$2x - 3y = 4,$	
③	$9x + 3y = 51$	(II) × 3
④	$11x = 55$	(I) + ③
⑤	$x = 5$	④ ÷ 11

Placing 5 for x in (II),

⑥	$15 + y = 17$	
⑦	$y = 2$	⑥ − 15

CHECK.

$$\text{(I)} \quad (2 \times 5) - (3 \times 2) \stackrel{?}{=} 4$$
$$10 - 6 = 4$$
$$\text{(II)} \quad (3 \times 5) + 2 \stackrel{?}{=} 17$$
$$15 + 2 = 17$$

Ans. $x = 5,\ y = 2.$

EXERCISES

Solve the following pairs of simultaneous equations, and check the answers.

1. $\begin{cases} x + 2y = 8, \\ 2x - y = 6. \end{cases}$
(See Ex. 1, page 200.)

2. $\begin{cases} x + y = 6, \\ x + 2y = 8. \end{cases}$
(See Ex. 1, page 200.)

3. $\begin{cases} x + y = 6, \\ x - y = 4. \end{cases}$
(See Ex. 2, page 200.)

4. $\begin{cases} 5a + 2b = 8, \\ 3a + 2b = 8. \end{cases}$

5. $\begin{cases} m + p = 7, \\ 2m + 3p = 17. \end{cases}$
(Multiply the first equation by 3.)

6. $\begin{cases} x + y = 7, \\ 2x + 3y = 17. \end{cases}$
(Multiply the first equation by 2.)

7. $\begin{cases} 5c + 6d = 16, \\ 3c + 4d = 10. \end{cases}$
(Eliminate d.)

8. $\begin{cases} 5p + 2w = 22, \\ 3p - 4w = 8. \end{cases}$
(Eliminate w.)

9. $\begin{cases} 4x + 9y = 5, \\ 2x + 3y = 2. \end{cases}$
(See Ex. 5, page 200.)

10. $\begin{cases} 6a + 3b = 12, \\ 4a - 5b = 8. \end{cases}$
(Eliminate b.)

11. $\begin{cases} 3x + 2y = 18, \\ 4x - y = 12. \end{cases}$
(Eliminate y and get values to nearest third figure.)

12. $\begin{cases} 5a + 4b = 15, \\ 2a - b = 4. \end{cases}$
(Eliminate b and get values to nearest third figure.)

13. The perimeter of a rectangle is 184 feet, and the length is 8 feet more than the width. Find its length and width.

SOLUTION. Let l = the length of the rectangle, and w = the width. The equations are:

(I) $l - w = 8$

(II) $2l + 2w = 184$

14. The perimeter of a rectangle is 240 feet; its length exceeds its width by 14 feet. Find its dimensions.

15. A farmer paid 5 men and 4 boys \$16 for a day's work. Later he paid 2 men and 6 boys \$13 for a day's work. How much did he pay each man per day? How much did he pay each boy?

16. For an entertainment, tickets were sold at 50 cents and 35 cents. The total proceeds were \$309 for 690 tickets. How many of each kind were sold?

17. For the same entertainment the next night the total proceeds were \$359.45 for 820 tickets. How many of each kind were sold?

18. A grocer has two kinds of tea, one selling at 40 cents per pound and the other at 50 cents. How many pounds of each kind must be used to make a mixture of 5 pounds to sell for \$2.20?

19. Milk is sold at 10 cents per quart and cream at 50 cents per quart. How many quarts of each will be needed to make a mixture of 12 quarts to sell for \$2?

20. At the same prices as in Ex. 19, how much of each will be needed to make 15 quarts to sell for \$2.50?

21. A grocer has two kinds of coffee, one selling at 22 cents per pound and the other at 32 cents. How many pounds of each kind must be used to make 6 pounds to sell for \$1.60?

22. At the same prices as in Ex. 21, how many pounds of each kind must be used to make 10 pounds to sell at 26 cents per pound?

23. Of two kinds of rice, one selling at 6 cents per pound and the other at 9 cents, how many pounds of each will make 30 pounds to sell at 8 cents per pound?

24. At the same prices as in Ex. 23, how many pounds of each kind must be used to make 18 pounds to sell at $6\frac{1}{2}$ cents per pound?

25. A mixture of 7-cent rice and 11-cent rice is to be sold at 3 pounds for a quarter. How many pounds of each kind must be used to make up forty 3-pound packages?

26. A store offers 10 pounds of sugar and two pounds of tea for \$1.55, or 6 pounds of sugar and one pound of tea for 85 cents. What is the selling price of each per pound?

27. A store offers one dozen cakes of laundry soap and one half dozen cakes of toilet soap for 90 cents, or 4 cakes of laundry soap and 3 of toilet soap for 38 cents. Find the cost of each kind per cake.

28. Nougatines selling at 40 cents per pound are to be mixed with chocolate almonds selling at 60 cents per pound to make a mixture to sell at 45 cents per pound. If 10 pounds of the mixture are wanted, how many pounds of each must be used?

29. English walnuts selling at 22 cents per pound are mixed with other nuts selling at 16 cents per pound, so as to get nuts to sell at 18 cents per pound. If the mixture made weighs $16\frac{1}{2}$ pounds, how many pounds of each are used?

30. In a certain shop 5 men and 3 boy-apprentices earn an average daily wage of \$2.25; on the same kind of work 7 men and 5 boy-apprentices earn an average daily wage of $\$2.16\frac{2}{3}$. Find the daily wage of each.

APPENDIX

REFERENCE-TABLES

1. Linear Measure

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards, or 16½ feet = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (mi.).
6 feet = 1 fathom.
1.151 miles = 1 knot.

2. Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile.
36 square miles = 1 township.

3. Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
16 cubic feet = 1 cord foot (cd. ft.).
128 cubic feet = 1 cord (cd.).

4. Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

5. Liquid Measure

4 gills (gi.) = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon (gal.).
63 gallons = 1 hogshead.

6. Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).
32 pounds = 1 bushel of oats.
48 pounds = 1 bushel of barley.
56 pounds = 1 bushel of corn or rye.
60 pounds = 1 bushel of wheat or potatoes.
196 pounds = 1 barrel (bbl.) of flour.
2240 pounds = 1 long ton of coal.

7. United States Money

10 mills (m.) = 1 cent (ct. or ¢).
10 cents = 1 dime (di.).
10 dimes = 1 dollar ($).
100 cents = 1 dollar.
10 dollars = 1 eagle.

8. Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
365 days = 1 common year (yr.).
366 days = 1 leap year.
12 months = 1 year.
360 days = 1 commercial year.
100 years = 1 century.

9. Counting

12 units = 1 dozen (doz.).
12 dozen = 1 gross (gro.).
24 sheets of paper = 1 quire.
500 sheets = 1 ream.

10. Circular Measure

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circumference.
$69\frac{1}{6}$ miles = 1 degree of latitude.

EQUIVALENTS

1 bushel = 2150 cu. in., or $\frac{5}{4}$ cu. ft. (approx.).
1 gallon = 231 cu. in.
1 cu. ft. = 7.48 gal., or $7\frac{1}{2}$ gal. (approx.).
1 cu. ft. water = 62.5 lb. (approx.).
1 ton of hay = 500 cu. ft. (approx.).
1 ton of coal = 35 cu. ft. (approx.).
π = 3.14159 (3.14 to *three* figures).
Foreign Money Equivalents. (See page 114.)

FORMULAS

I. Areas and Perimeters of Plane Figures

A. Notation Used in Formulas

A = area (in square units).
b, b_1, b_2 = bases
h = height or altitude
s = side
p = perimeter
c = circumference
d = diameter
r = radius
} (in linear units).

B. Formulas

1.	Perimeter of a rectangle,	$p = 2b + 2h$,	p. 126
2.	Perimeter of a square,	$p = 4s$,	p. 127
3.	Area of a rectangle,	$A = bh$,	p. 128
4.	Area of a square,	$A = s^2$,	p. 131
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6.	Area of a triangle,	$A = \frac{1}{2}bh$,	p. 136
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8.	Circumference of a circle,	$c = \pi d$ or $2\pi r$,	p. 149
9.	Area of a circle,	$A = \pi r^2$,	p. 149

II. Surfaces and Volumes of Solids

A. Notation Used in Formulas

V = volume (in cubic units).
B = area of base (in square units).
S = area of curved surface (in square units).
l = length of block or slant height of cone
w = width
h = height
e = edge
r = radius
(in linear units).

B. Formulas

1.	Volume of a block,	$V = lwh$,	p. 154
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8.	Volume of a cylinder,	$V = \pi r^2 h$,	p. 163
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III. Other Formulas

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6.	Percentage,	$p = rb$,	p. 38

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